

$$U_y = \exp(-iH y)$$

$$\text{Var}[y] \Delta^2 H_{\text{tot}} \geq \frac{1}{4M}$$

generalised Heisenberg relation

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}



How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}

???

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}



How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}



$$P(y|\vec{x})$$

$$P(y)$$

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}

Bayes' theorem

$$P(y|\vec{x}) = \frac{1}{P(\vec{x})} P(\vec{x}|y) P(y)$$

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}

Bayes' theorem

$$P(y|\vec{x}) = \frac{1}{P(\vec{x})} P(\vec{x}|y) P(y)$$

$$P(\vec{x}|y) = p(x_1|y)p(x_2|y)\dots p(x_M|y)$$

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}

Bayesian estimators

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}

???

Bayesian estimators

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$

y_{est}

Bayesian estimators

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$



Bayesian estimators

$$y_{\text{est}} = \int y P(y|\vec{x}) dy$$

How do we get an estimator?

$x_1, x_2, x_3, \dots, x_M$



Bayesian estimators

$$y_{\text{est}} = \int y P(y|\vec{x}) dy$$

$$\text{Var}[y] = \int (y - y_{\text{est}})^2 P(y|\vec{x}) dy$$

Multiparameter regime

Multiparameter regime



Multiparameter regime



Fisher information

$$\mathcal{I}_{j,k}(\vec{y}) = \int \frac{\partial_j p(x|\vec{y}) \partial_k p(x|\vec{y})}{p(x|\vec{y})} dx$$

Fisher information

$$\mathcal{I}_{j,k}(\vec{y}) = \int \frac{\partial_j p(x|\vec{y}) \partial_k p(x|\vec{y})}{p(x|\vec{y})} dx$$

$$\text{Cov}[\vec{y}]_{j,k} = \mathbf{E}[(y_j - y_{j,\text{est}})(y_k - y_{k,\text{est}})]$$

Fisher information

$$\mathcal{I}_{j,k}(\vec{y}) = \int \frac{\partial_j p(x|\vec{y}) \partial_k p(x|\vec{y})}{p(x|\vec{y})} dx$$

$$\text{Cov}[\vec{y}] \geq \frac{1}{M} \underline{\mathcal{I}}(\vec{y})^{-1}$$

multiparameter Cramér–Rao bound

Fisher information

$$\mathcal{I}_{j,k}(\vec{y}) = \int \frac{\partial_j p(x|\vec{y}) \partial_k p(x|\vec{y})}{p(x|\vec{y})} dx$$

[lots of maths which is essentially a cleverer use of the Cauchy Schwartz inequality than before]

$$\text{Cov}[\vec{y}] \geq \frac{1}{M} \underline{\mathcal{I}}(\vec{y})^{-1}$$

multiparameter Cramér–Rao bound

measure in the direction in which the state curves the most

we look at the curvature parameter by parameter

$$2\partial_k \rho_{\vec{y}} = L_k \rho_{\vec{y}} + \rho_{\vec{y}} L_k$$

we look at the curvature parameter by parameter

$$2\partial_k \rho_{\vec{y}} = L_k \rho_{\vec{y}} + \rho_{\vec{y}} L_k$$

$$\mathcal{Q}_{j,k}(\vec{y}) = \frac{1}{2} \text{Tr}[\rho_{\vec{y}}(L_j L_k + L_k L_j)]$$

Quantum Fisher Information

we look at the curvature parameter by parameter

$$2\partial_k \rho_{\vec{y}} = L_k \rho_{\vec{y}} + \rho_{\vec{y}} L_k$$

$$\mathcal{Q}_{j,k}(\vec{y}) = \frac{1}{2} \text{Tr}[\rho_{\vec{y}}(L_j L_k + L_k L_j)]$$

Quantum Fisher Information

$$\underline{\mathcal{Q}}(\vec{y}) \geq \underline{\mathcal{I}}(\vec{y})$$

Quantum Cramér-Rao bound

we look at the curvature parameter by parameter

$$2\partial_k \rho_{\vec{y}} = L_k \rho_{\vec{y}} + \rho_{\vec{y}} L_k$$

$$\underline{\mathcal{Q}}(\vec{y}) \geq \underline{\mathcal{I}}(\vec{y})$$

Quantum Cramér-Rao bound

we look at the curvature parameter by parameter

$$2\partial_k \rho_{\vec{y}} = L_k \rho_{\vec{y}} + \rho_{\vec{y}} L_k$$

$$\underline{\mathcal{Q}}(\vec{y}) \geq \underline{\mathcal{I}}(\vec{y})$$

Quantum Cramér-Rao bound

The optimal measurement is unknown

compatibility conditions

compatibility conditions

$\text{Tr}[\rho_{\vec{y}}\{L_j, L_k\}] \neq 0$ though luck

compatibility conditions

$\text{Tr}[\rho_{\vec{y}}\{L_j, L_k\}] \neq 0$ though luck

$\text{Tr}[\rho_{\vec{y}}\{L_j, L_k\}] = 0$ joint optimal
estimability

compatibility conditions

$\text{Tr}[\rho_{\vec{y}}\{L_j, L_k\}] \neq 0$ though Luck

$\text{Tr}[\rho_{\vec{y}}\{L_j, L_k\}] = 0$ joint optimal
estimability *

* using a collective measurement
on a large number of replicas

two parameters are enough

on closer inspection

$$\mathcal{I}_{1,1}^{\text{eff}} = \frac{1}{(\mathcal{I}^{-1})_{1,1}}$$

two parameters are enough

$$\text{Var}[y_1] \geq \frac{(\mathcal{I}^{-1})_{1,1}}{M}$$

$$\text{Var}[y_2] \geq \frac{(\mathcal{I}^{-1})_{2,2}}{M}$$

on closer inspection

$$\mathcal{I}_{1,1}^{\text{eff}} = \frac{1}{(\mathcal{I}^{-1})_{1,1}}$$

two parameters are enough

$$\text{Var}[y_1] \geq \frac{(\mathcal{I}^{-1})_{1,1}}{M}$$

$$\text{Var}[y_2] \geq \frac{(\mathcal{I}^{-1})_{2,2}}{M}$$

on closer inspection

$$\mathcal{I}_{1,1}^{\text{eff}} = \frac{1}{(\mathcal{I}^{-1})_{1,1}}$$

$$\mathcal{I}_{1,1}^{\text{eff}} = \mathcal{I}_{1,1} - \frac{(\mathcal{I}_{1,2})^2}{\mathcal{I}_{2,2}}$$

two parameters are enough

$$\text{Var}[y_1] \geq \frac{(\mathcal{I}^{-1})_{1,1}}{M}$$

$$\text{Var}[y_2] \geq \frac{(\mathcal{I}^{-1})_{2,2}}{M}$$

on closer inspection

$$\mathcal{I}_{1,1}^{\text{eff}} = \frac{1}{(\mathcal{I}^{-1})_{1,1}}$$

$$\mathcal{I}_{1,1}^{\text{eff}} = \cancel{\mathcal{I}_{1,1}} - \frac{(\mathcal{I}_{1,2})^2}{\mathcal{I}_{2,2}}$$

two parameters are enough

$$\text{Var}[y_1] \geq \frac{(\mathcal{I}^{-1})_{1,1}}{M}$$

$$\text{Var}[y_2] \geq \frac{(\mathcal{I}^{-1})_{2,2}}{M}$$

on closer inspection

$$\mathcal{I}_{1,1}^{\text{eff}} = \frac{1}{(\mathcal{I}^{-1})_{1,1}}$$

$$\mathcal{I}_{1,1}^{\text{eff}} = \cancel{\mathcal{I}_{1,1}} - \cancel{\frac{(\mathcal{I}_{1,2})^2}{\mathcal{I}_{2,2}}}$$

two parameters are enough

$$\text{Var}[y_1] \geq \frac{(\mathcal{I}^{-1})_{1,1}}{M}$$

$$\text{Var}[y_2] \geq \frac{(\mathcal{I}^{-1})_{2,2}}{M}$$

on closer inspection

$$\mathcal{I}_{1,1}^{\text{eff}} = \frac{1}{(\mathcal{I}^{-1})_{1,1}}$$

$$\mathcal{I}_{1,1}^{\text{eff}} = \cancel{\mathcal{I}_{1,1}} - \frac{(\mathcal{I}_{1,2})^2}{\cancel{\mathcal{I}_{2,2}}}$$

what is commonly assessed

what is commonly assessed

$$\Upsilon = \text{Tr}[\underline{\mathcal{I}}^{-1} \cdot \underline{Q}]$$

what is commonly assessed

$$\Upsilon = \text{Tr}[\underline{\mathcal{I}}^{-1} \cdot \underline{Q}] \leq K$$

what is commonly assessed

$$\Upsilon = \text{Tr}[\underline{\mathcal{I}}^{-1} \cdot \underline{Q}] \leq l$$

what is commonly assessed

$$\Upsilon = \text{Tr}[\underline{\mathcal{I}}^{-1} \cdot \underline{Q}] \leq l$$

the limit gets better by increasing the dimensionality

what is commonly assessed

$$\Upsilon = \text{Tr}[\underline{\mathcal{I}}^{-1} \cdot \underline{Q}] \leq l$$

information is there, but there might not
enough room to extract it!

what is commonly assessed

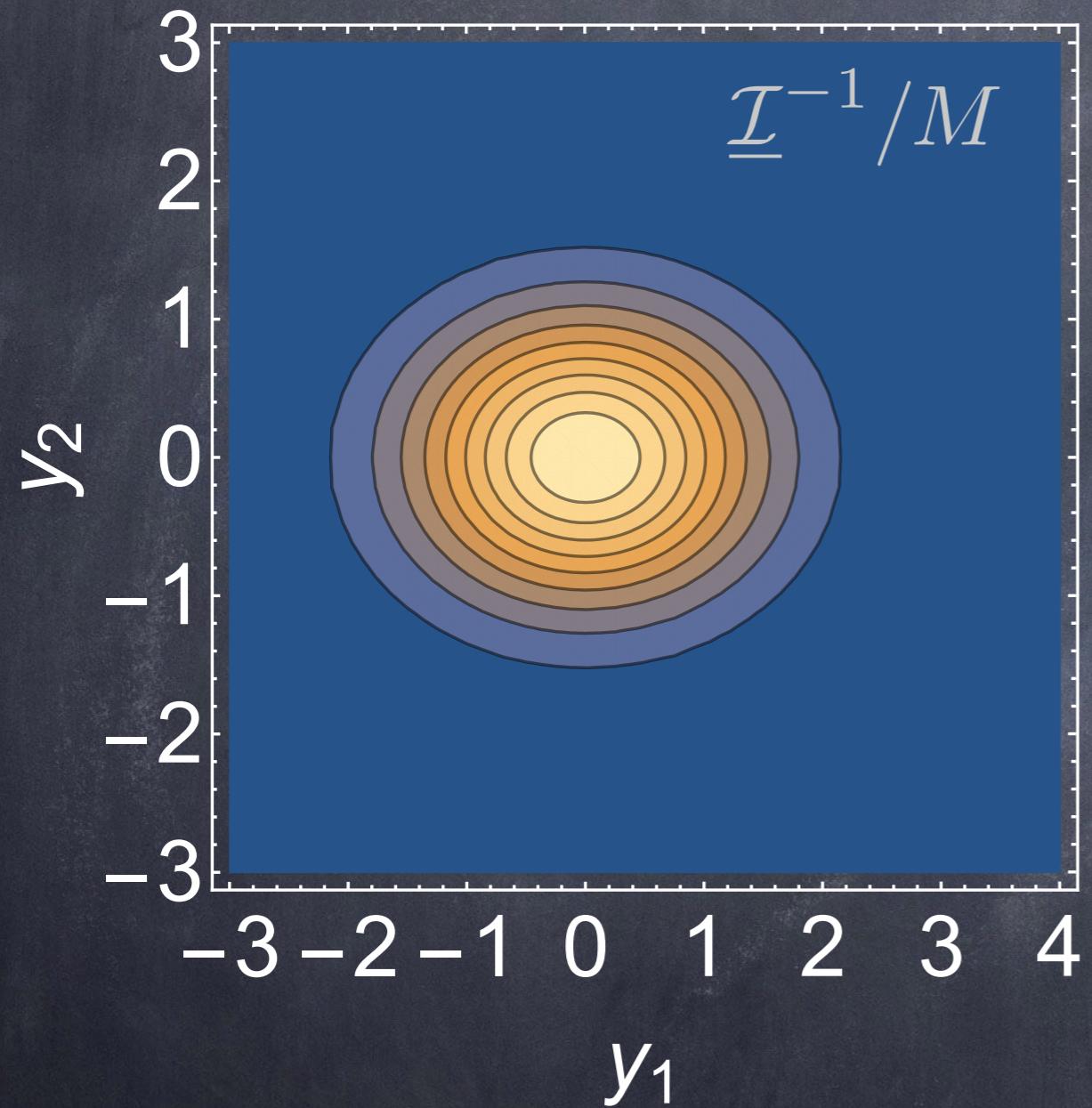
$$\Upsilon = \text{Tr}[\underline{\mathcal{I}}^{-1} \cdot \underline{Q}] \leq l$$

information is there, but there might not
enough room to extract it!

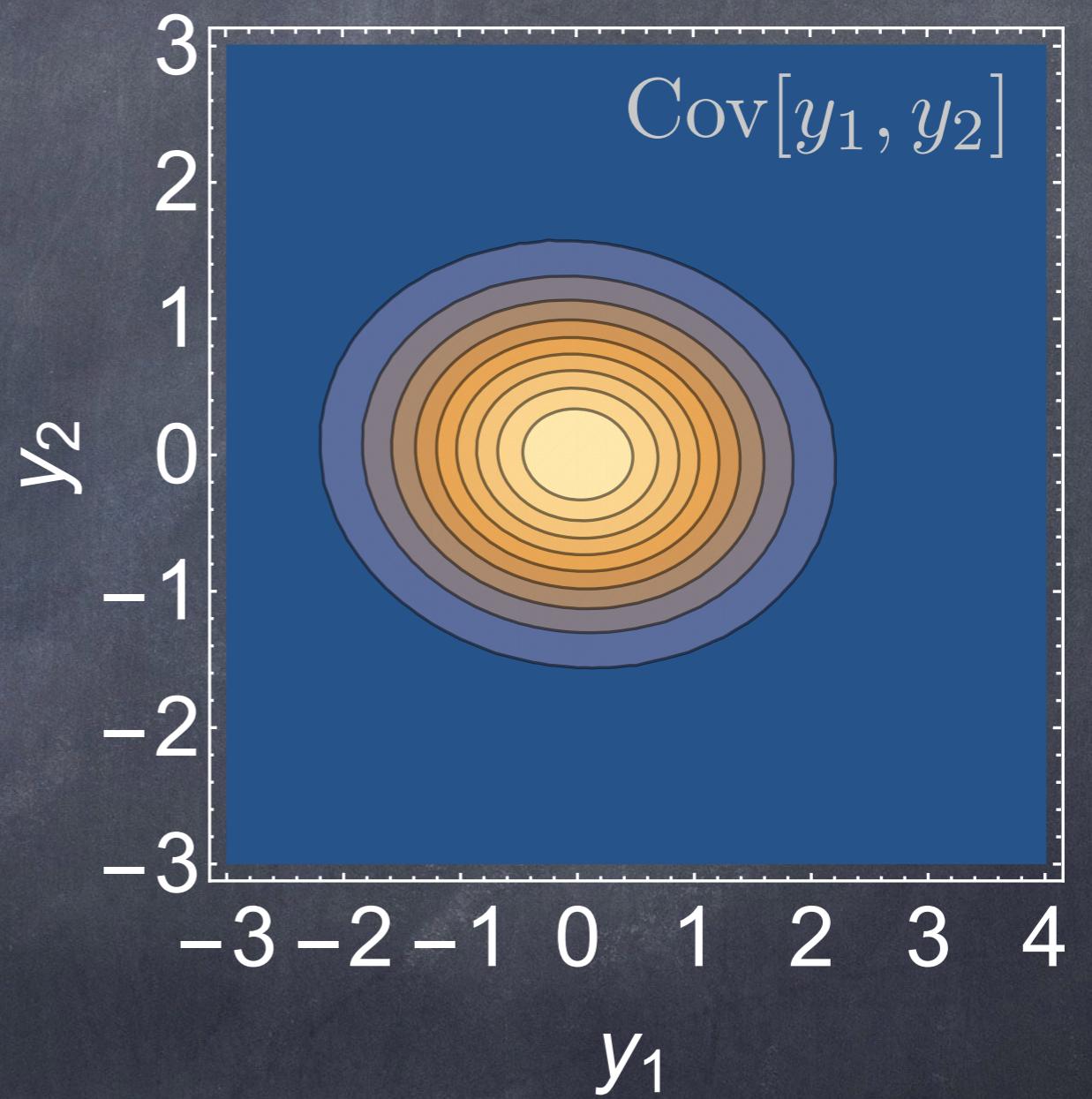
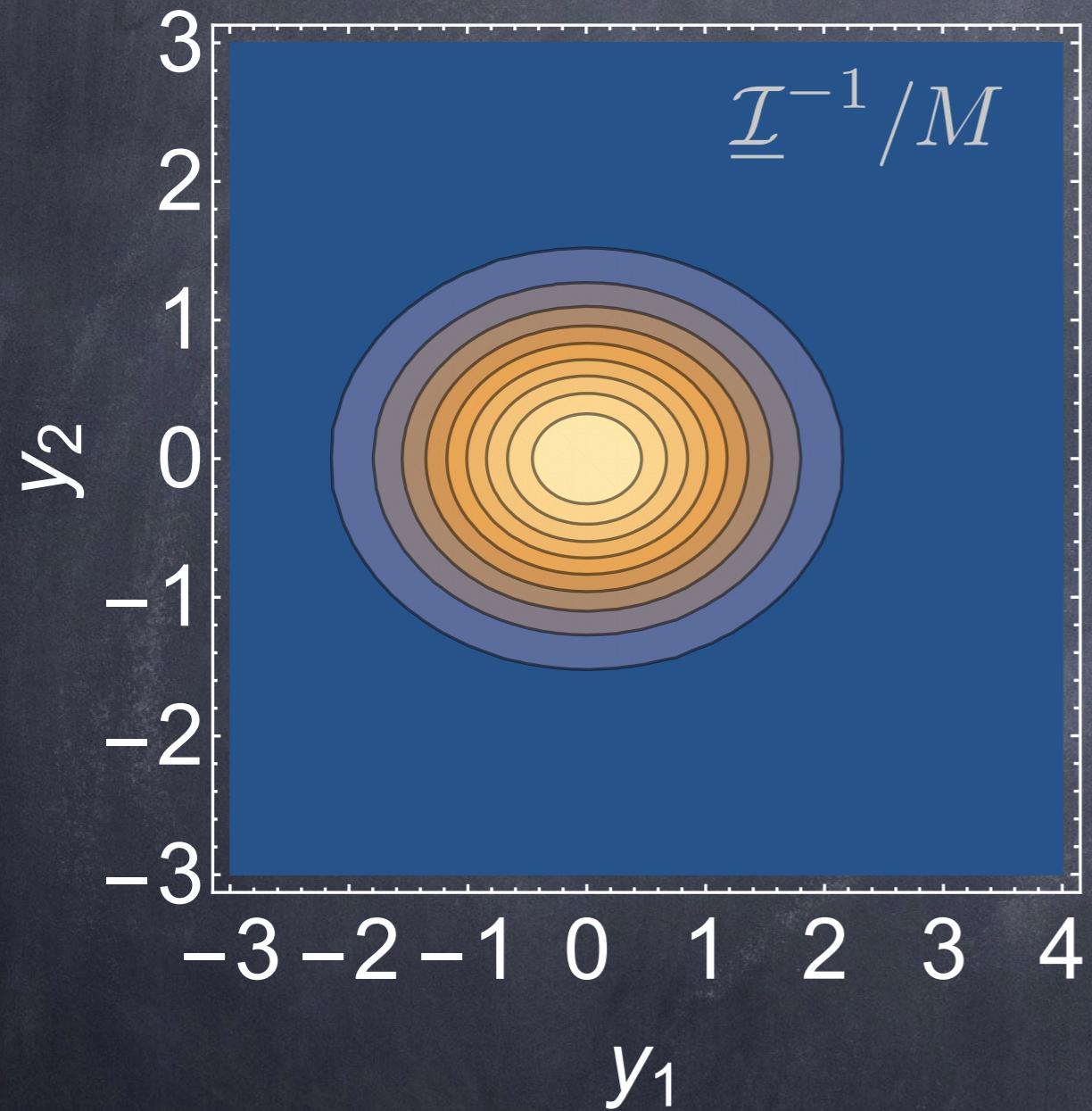
This is a measure of information
extraction (not of information itself)

comparing two covariance matrices

comparing two covariance matrices



comparing two covariance matrices



hypothesis testing!

hypothesis testing!

H_0 : the two
covariance
matrices are equal

hypothesis testing!

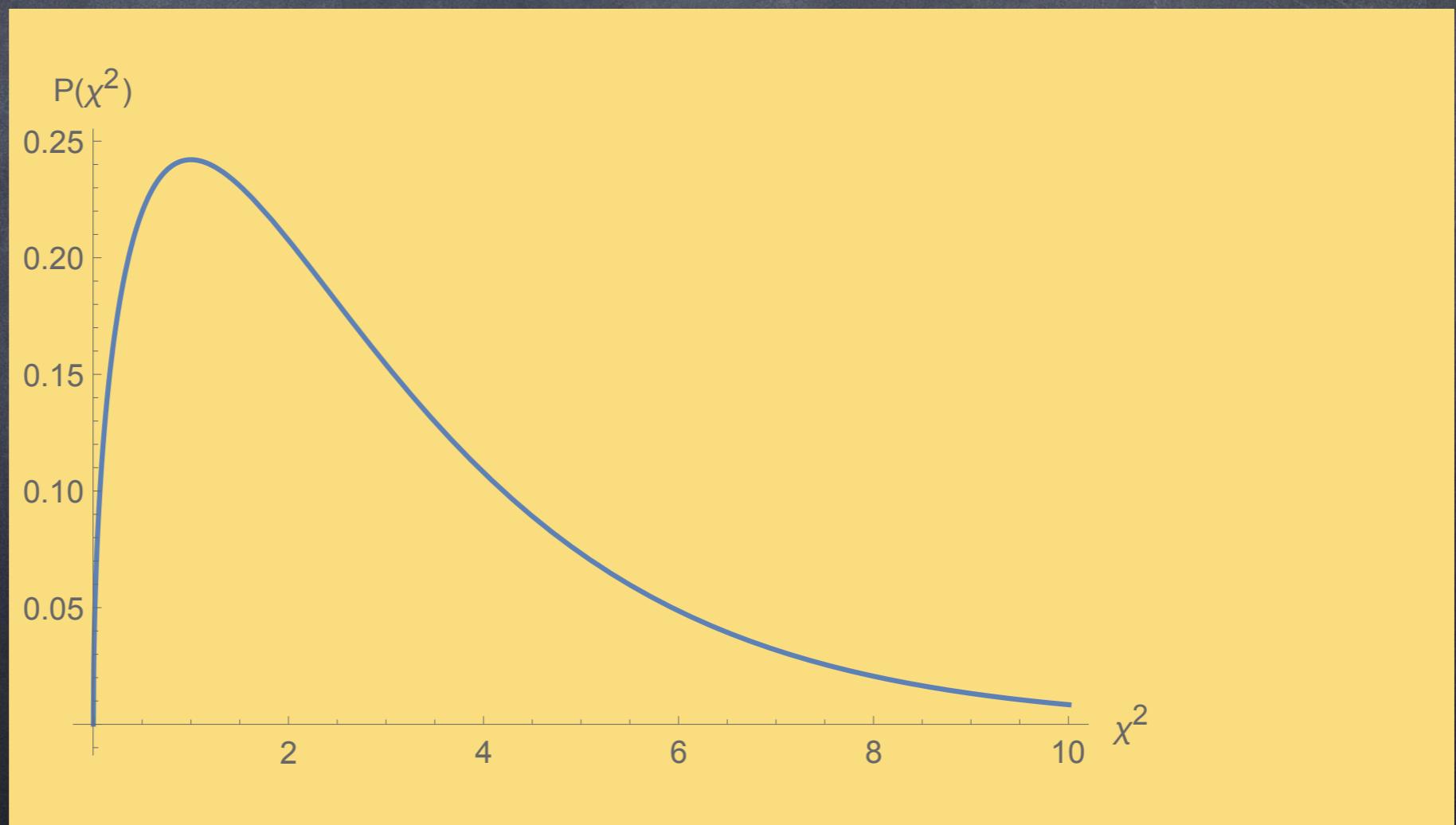
H_0 : the two
covariance
matrices are equal

H_1 : you're doing it
wrong

hypothesis testing!

H_0 : the two covariance matrices are equal

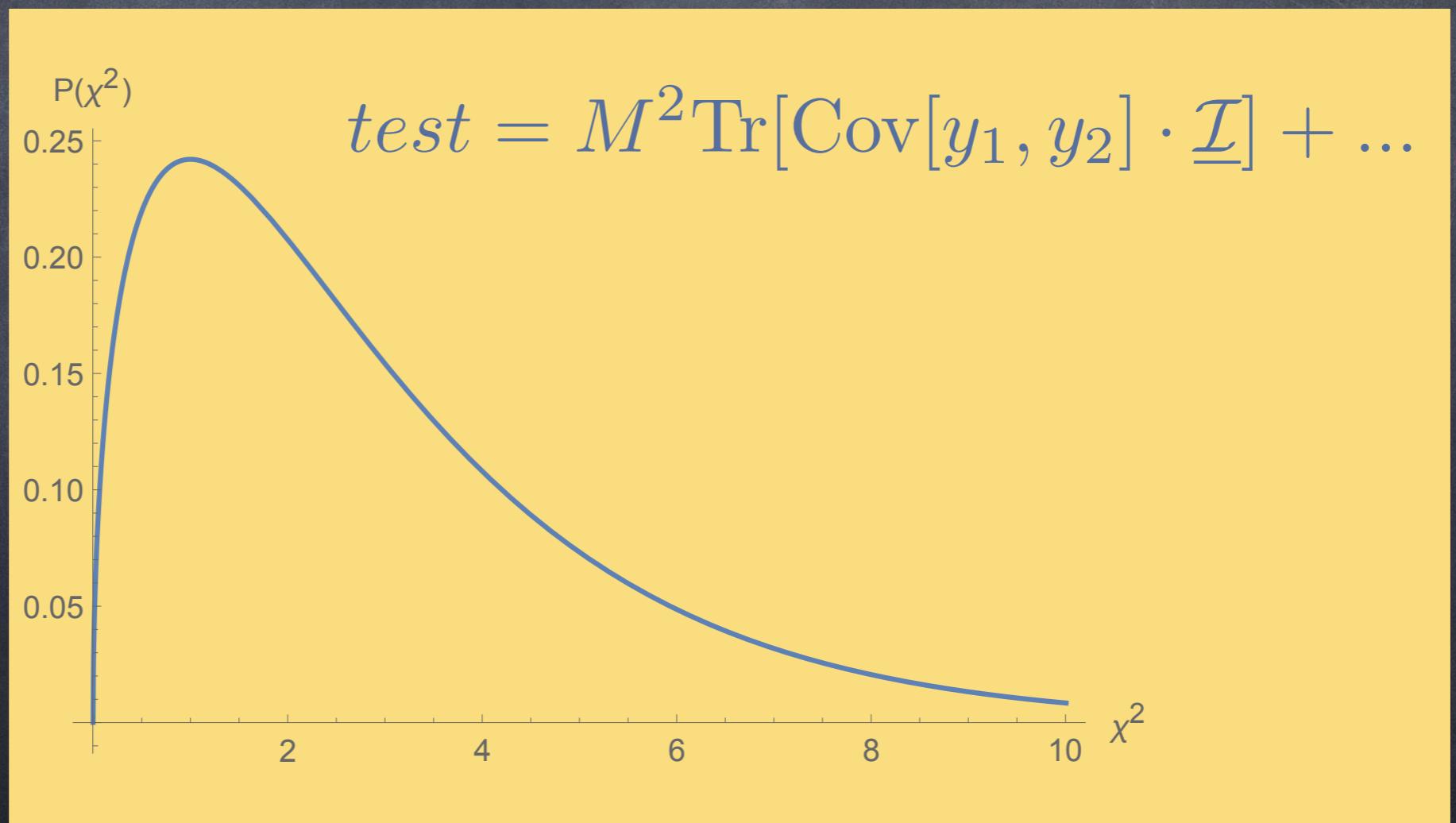
H_1 : you're doing it wrong



hypothesis testing!

H_0 : the two covariance matrices are equal

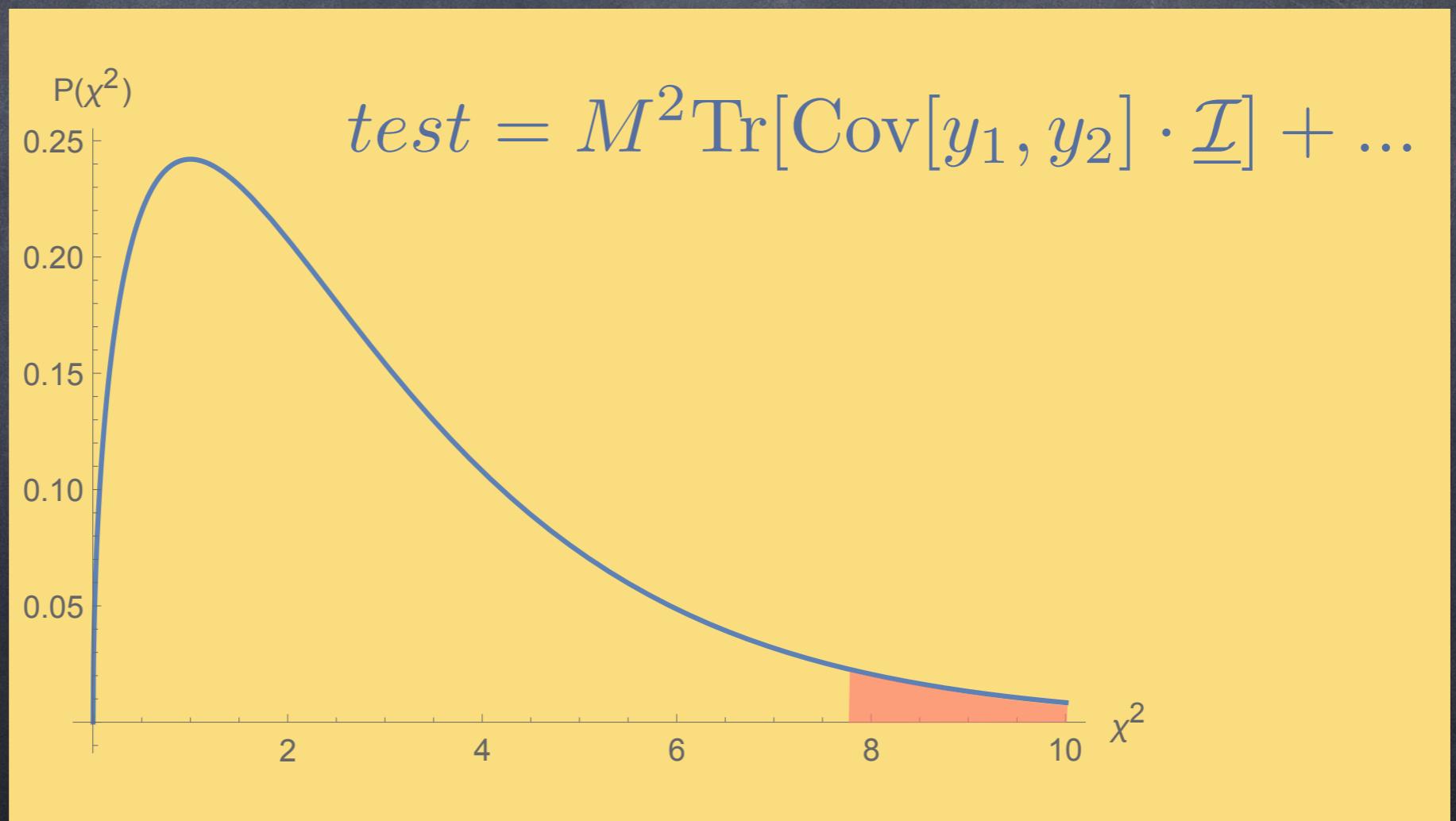
H_1 : you're doing it wrong



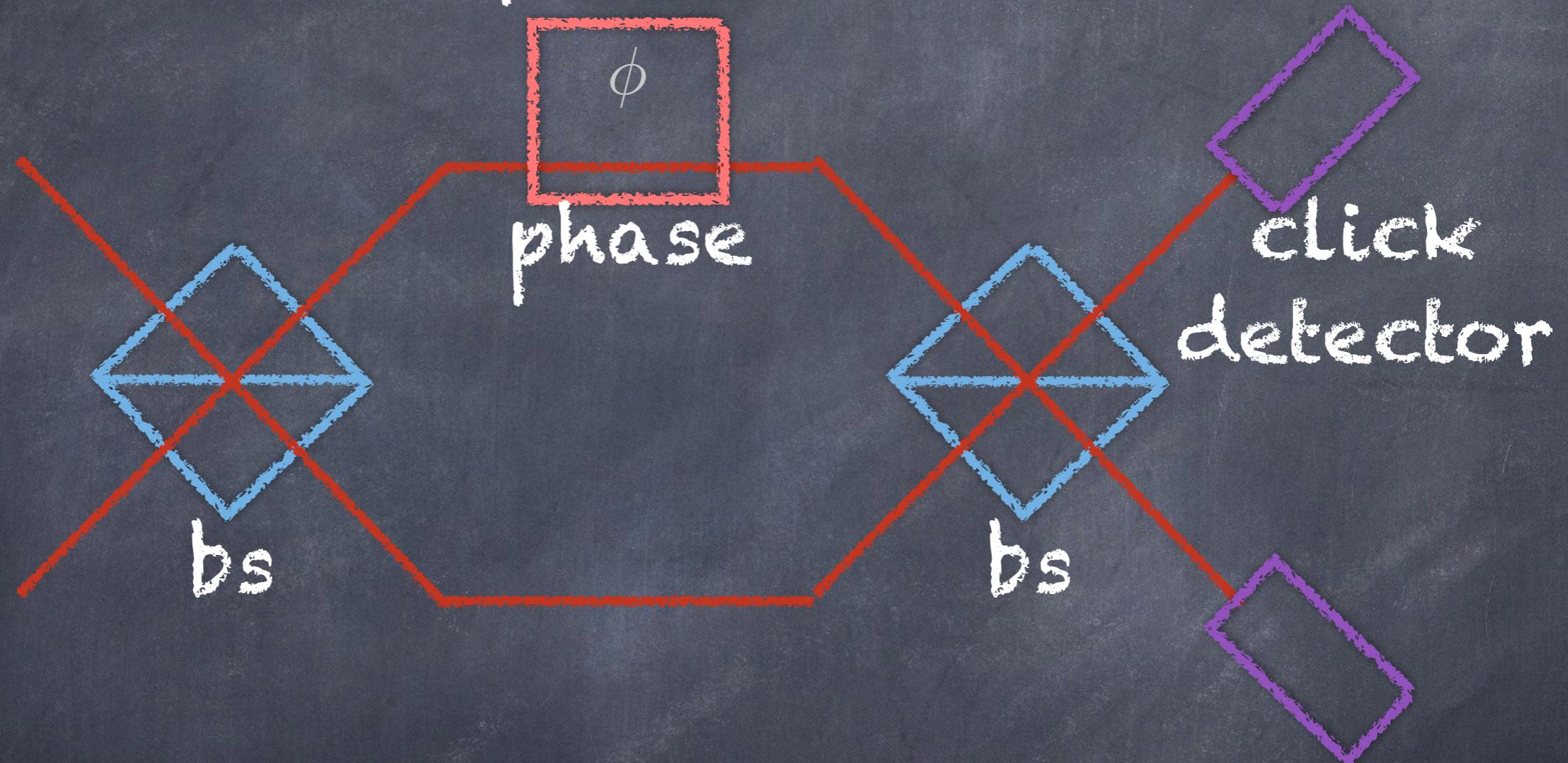
hypothesis testing!

H_0 : the two covariance matrices are equal

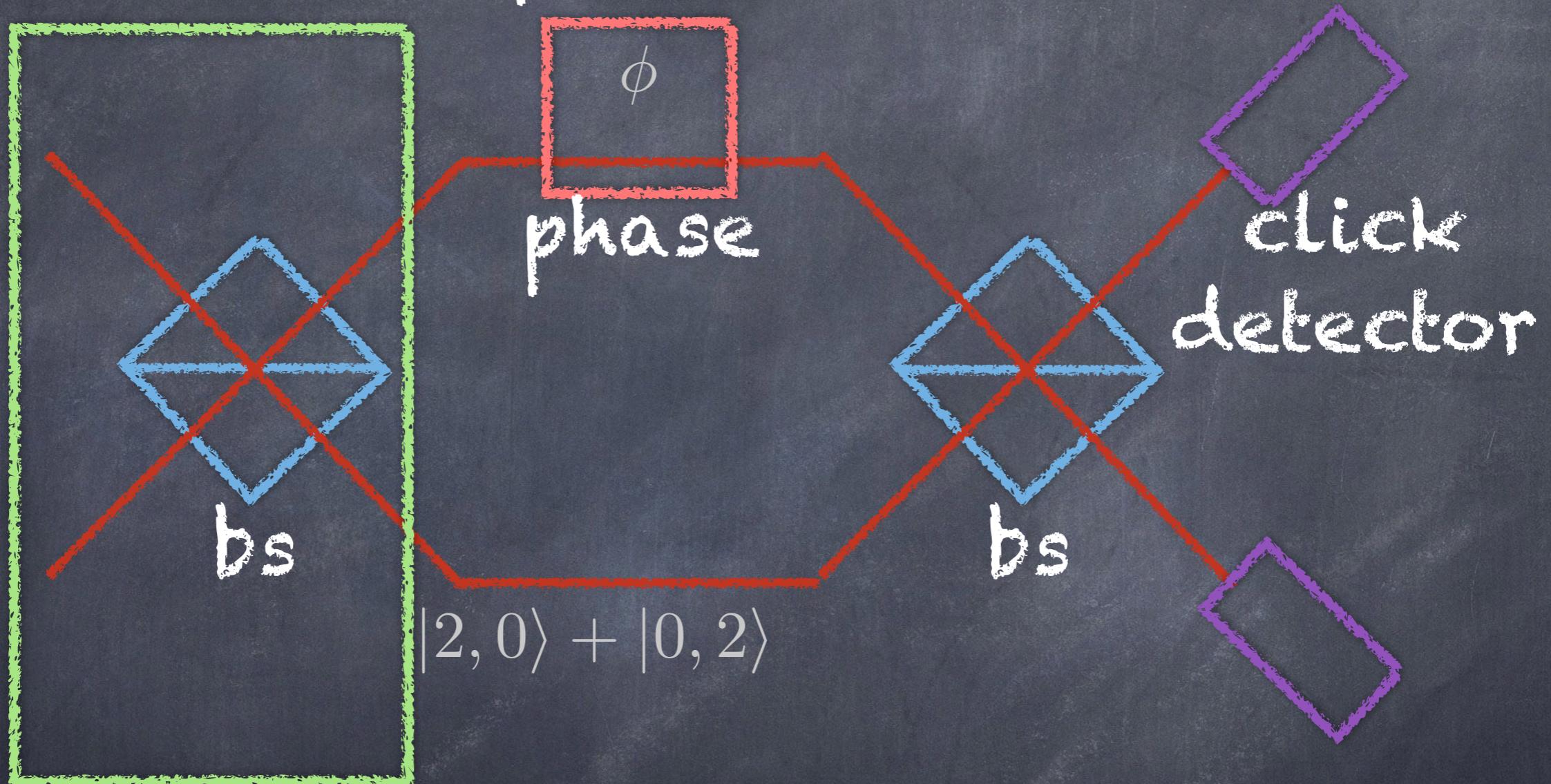
H_1 : you're doing it wrong



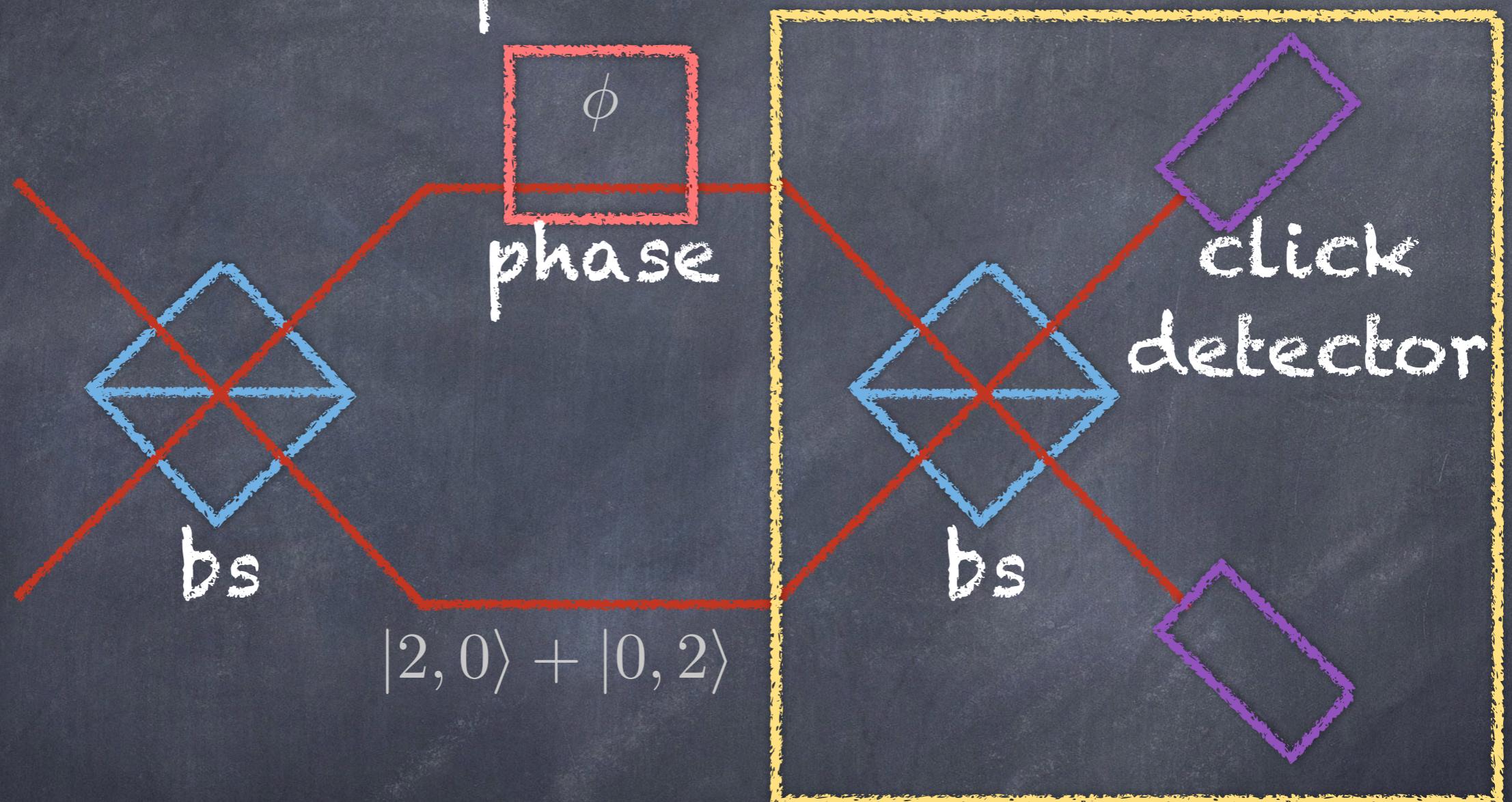
here's an example



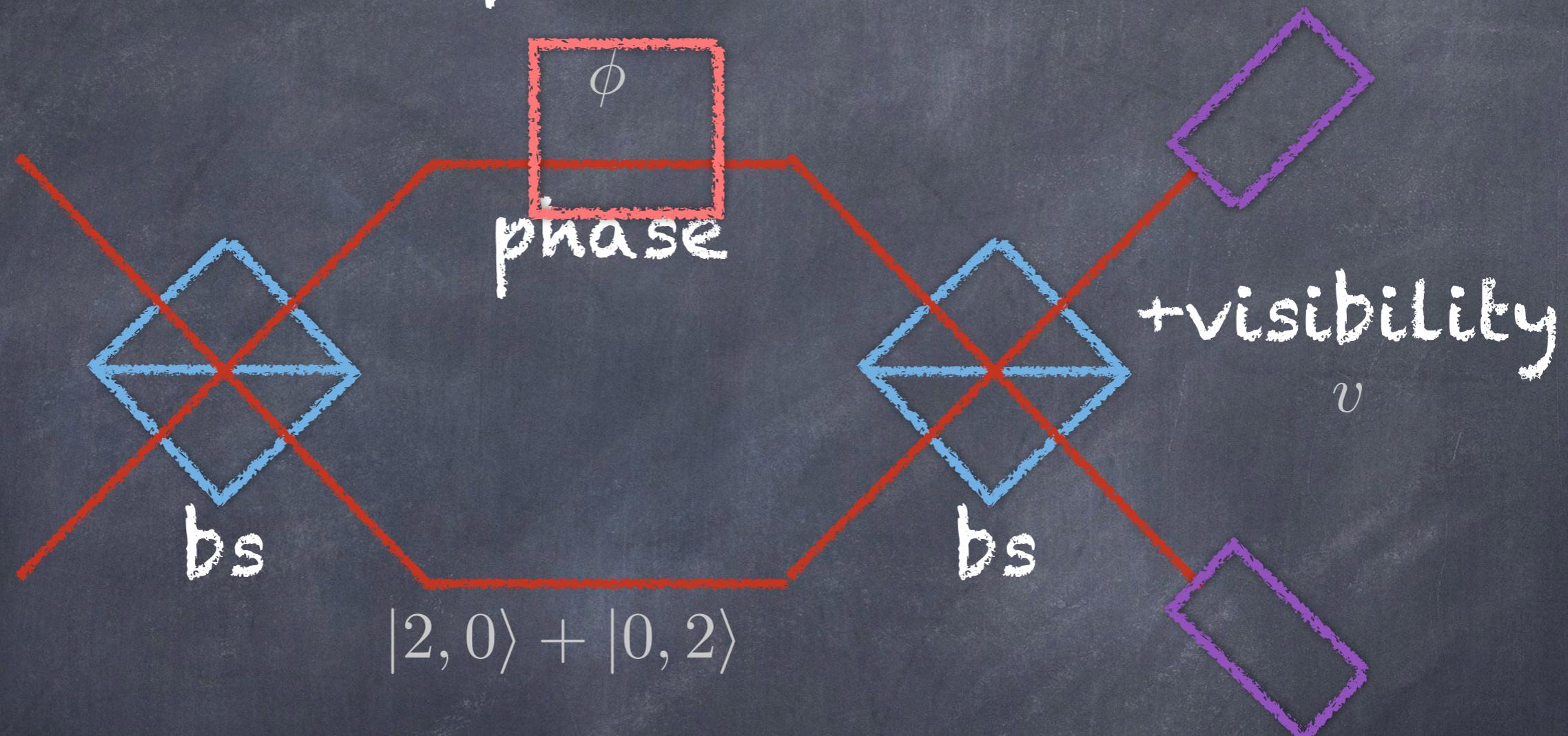
here's an example



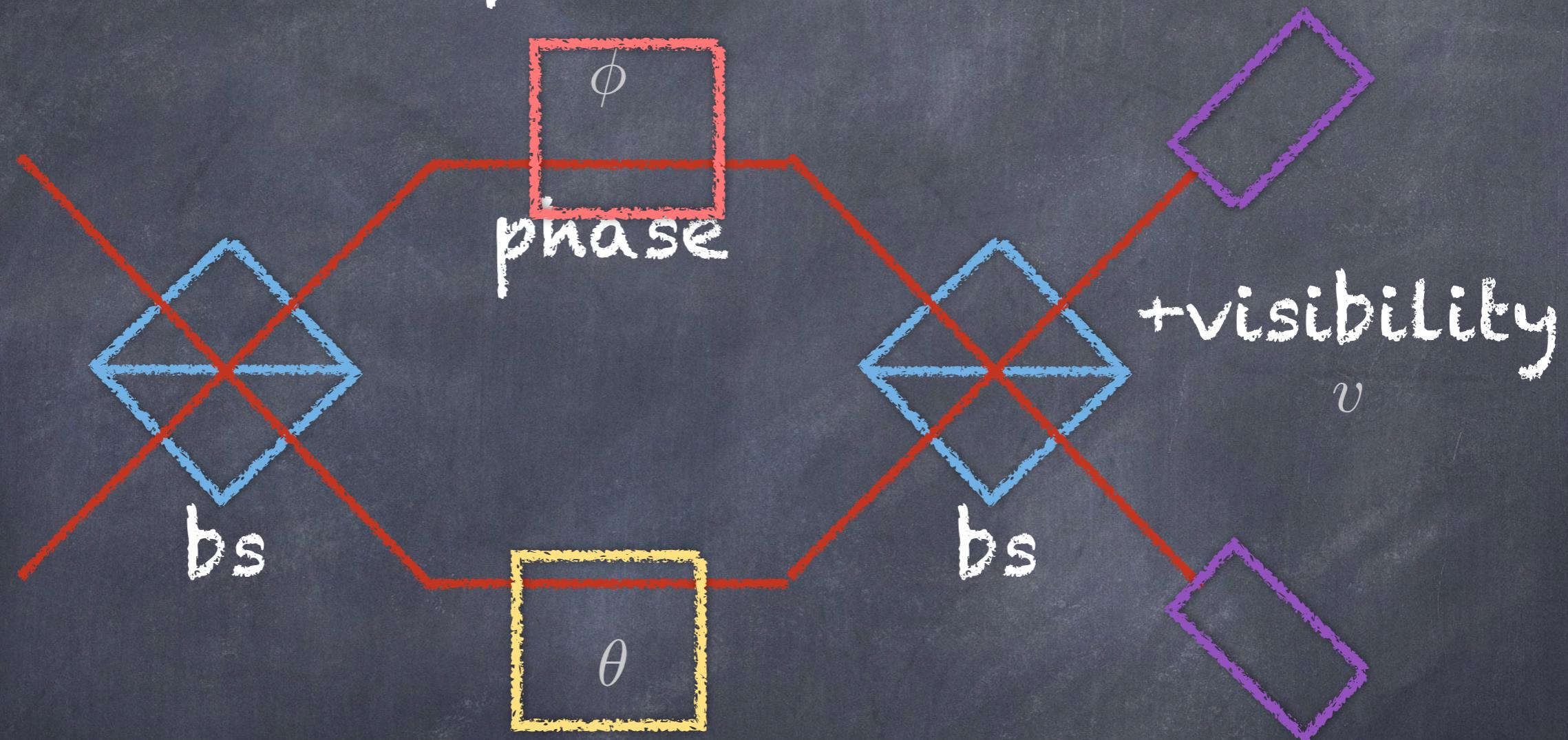
here's an example



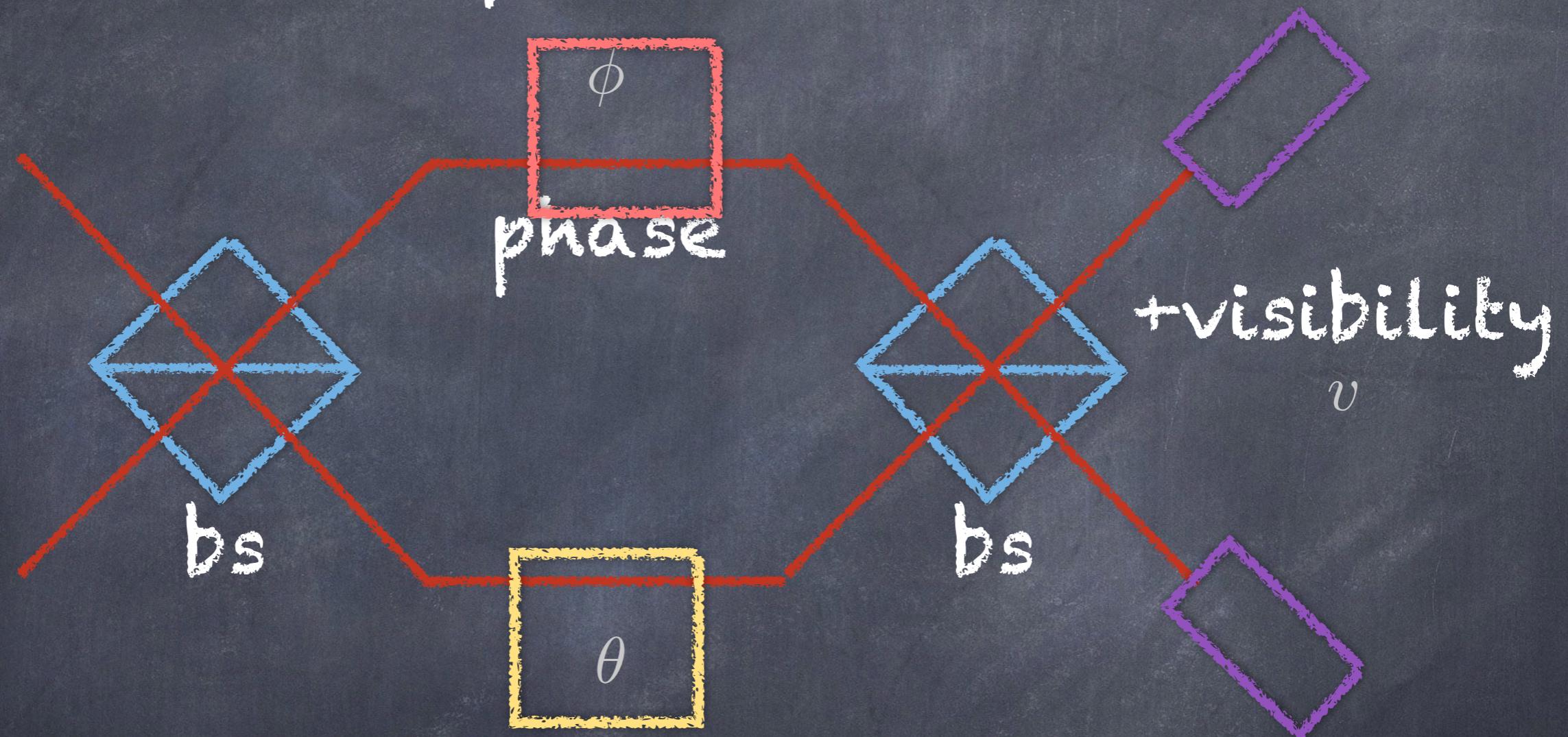
here's an example



here's an example



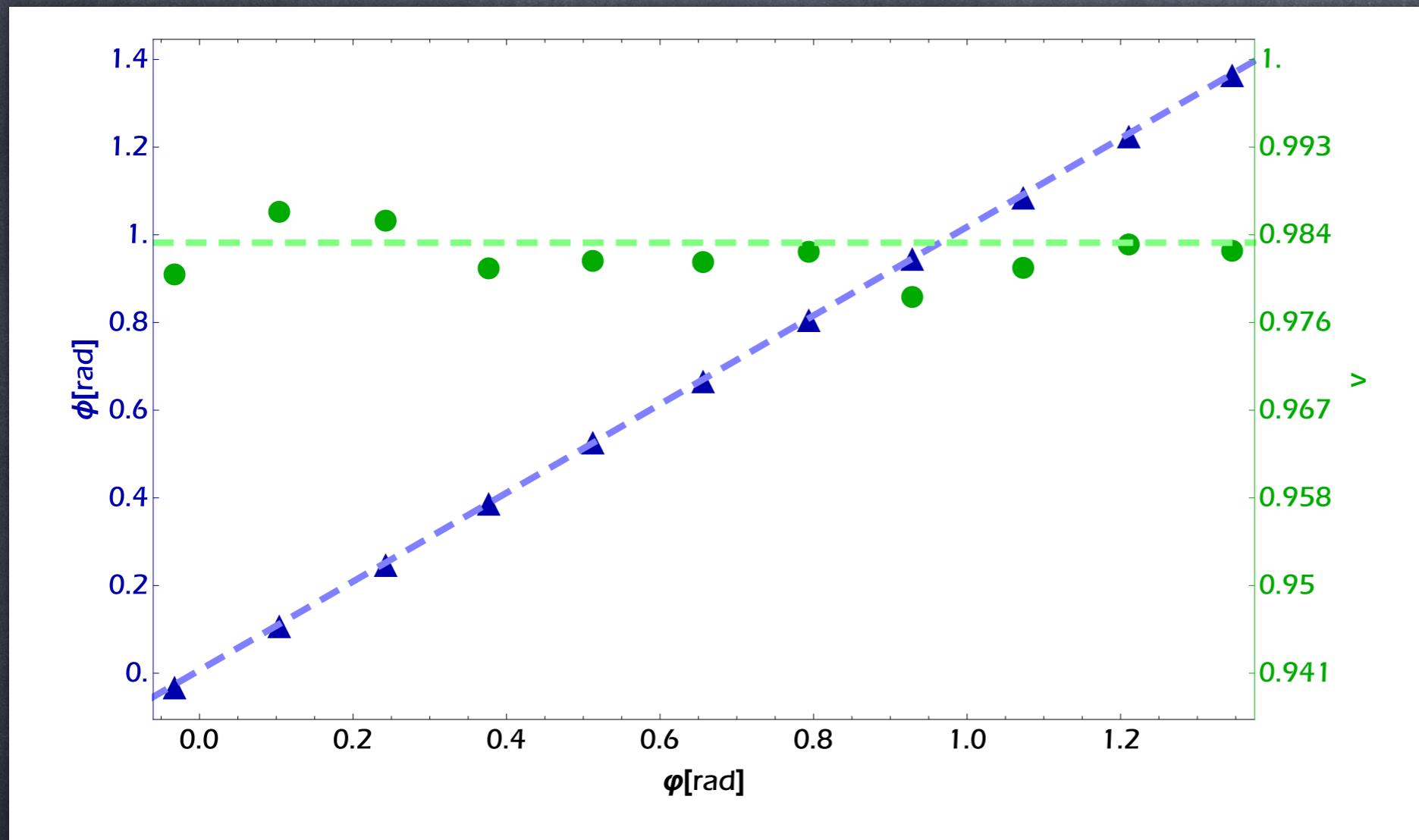
here's an example



$$p(\phi, v) \sim 1 + v \cos(\phi - \theta)$$

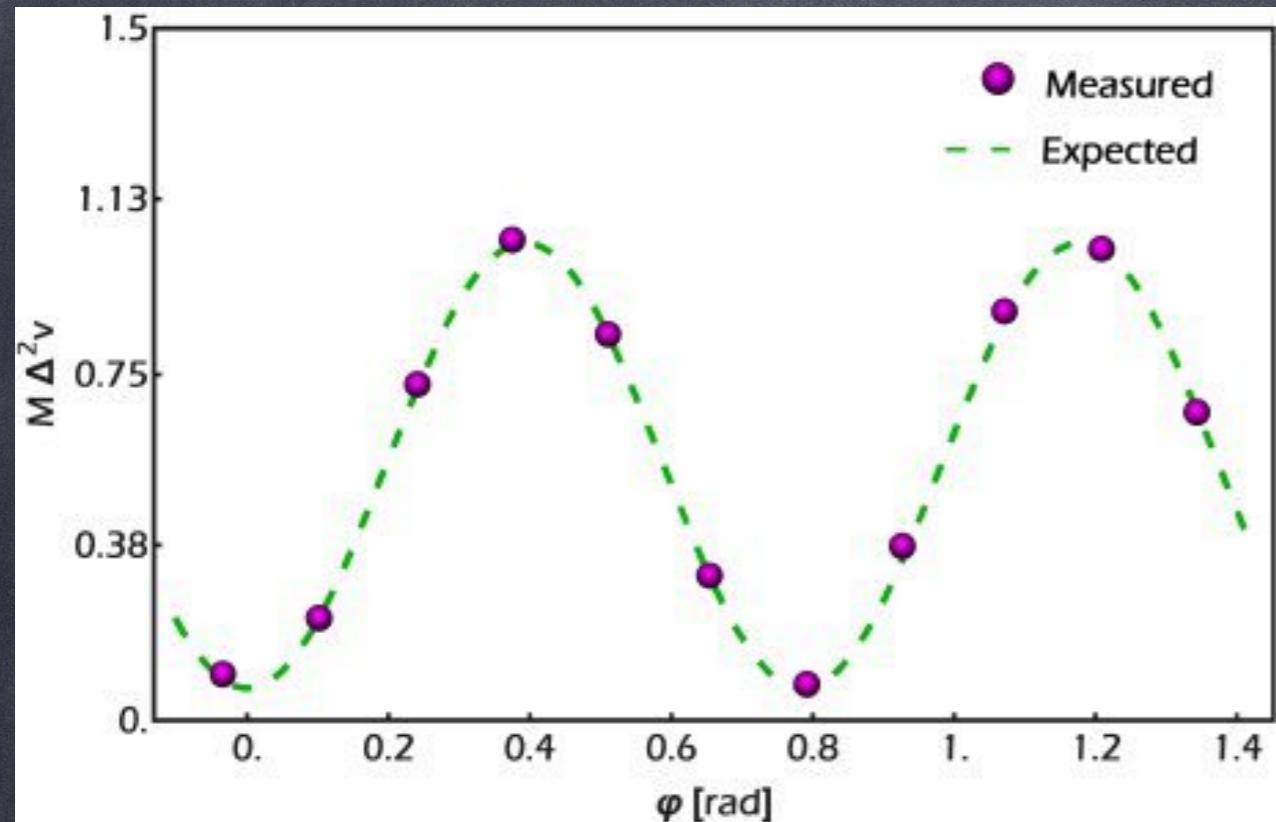
first: are there biases?

first: are there biases?

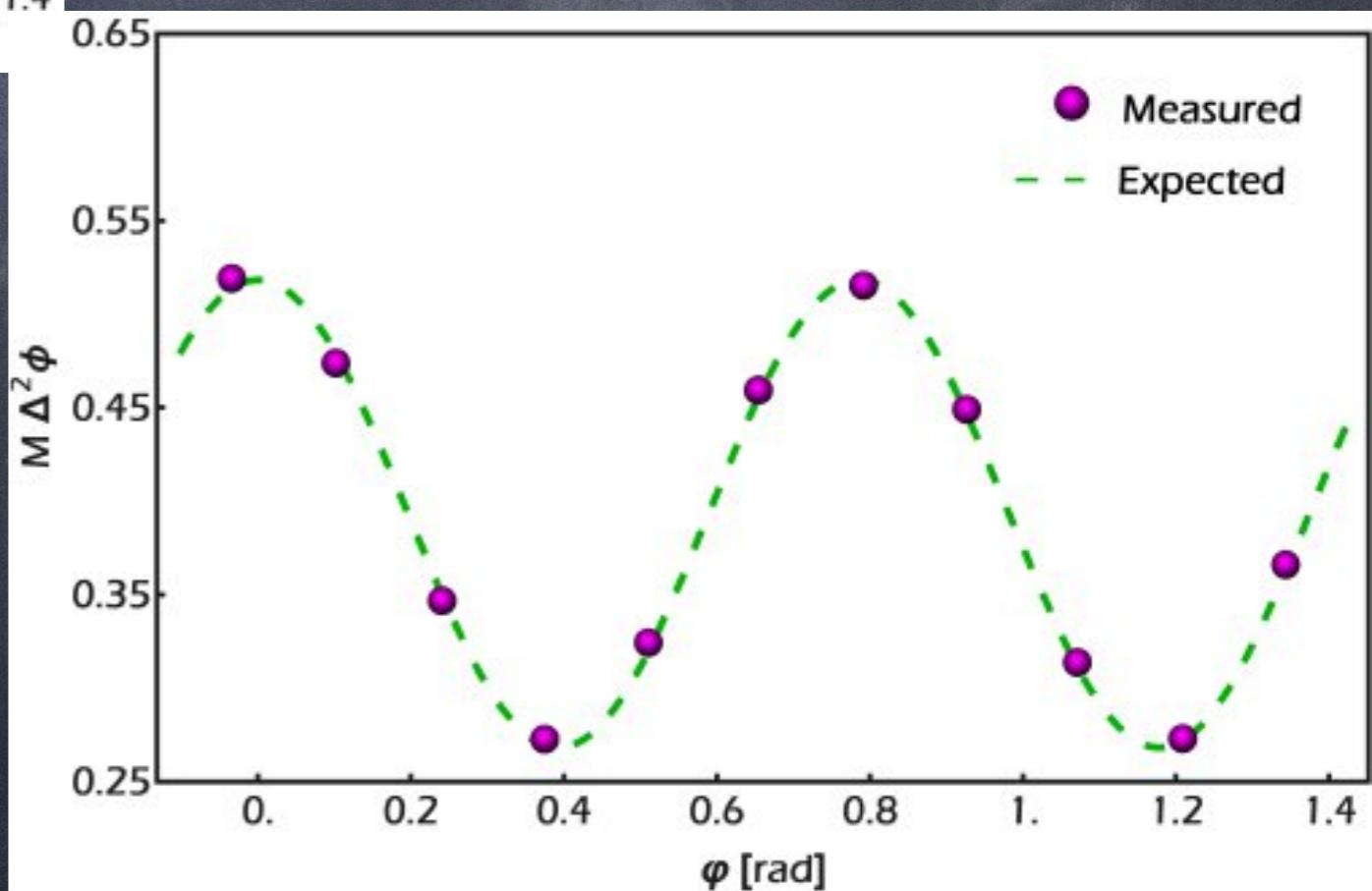
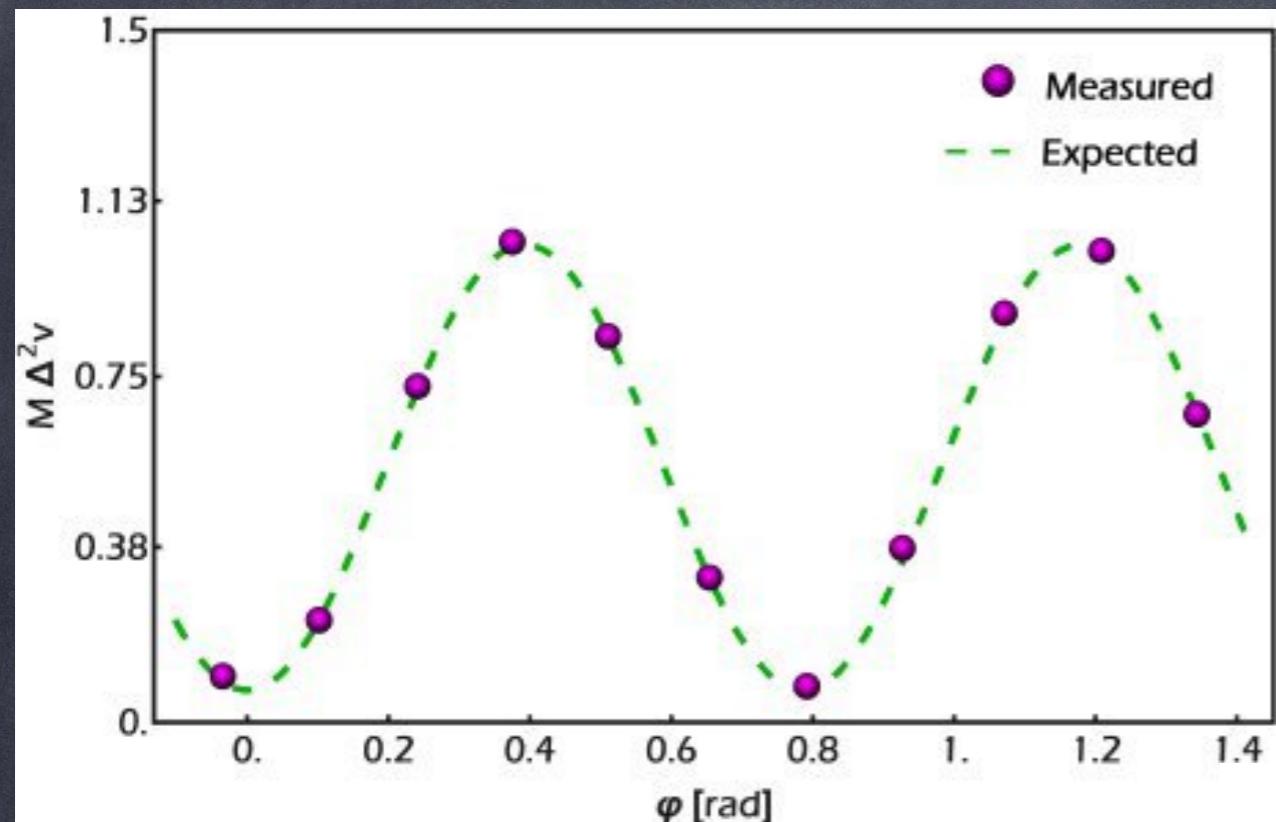


then: are we doing it right?

then: are we doing it right?

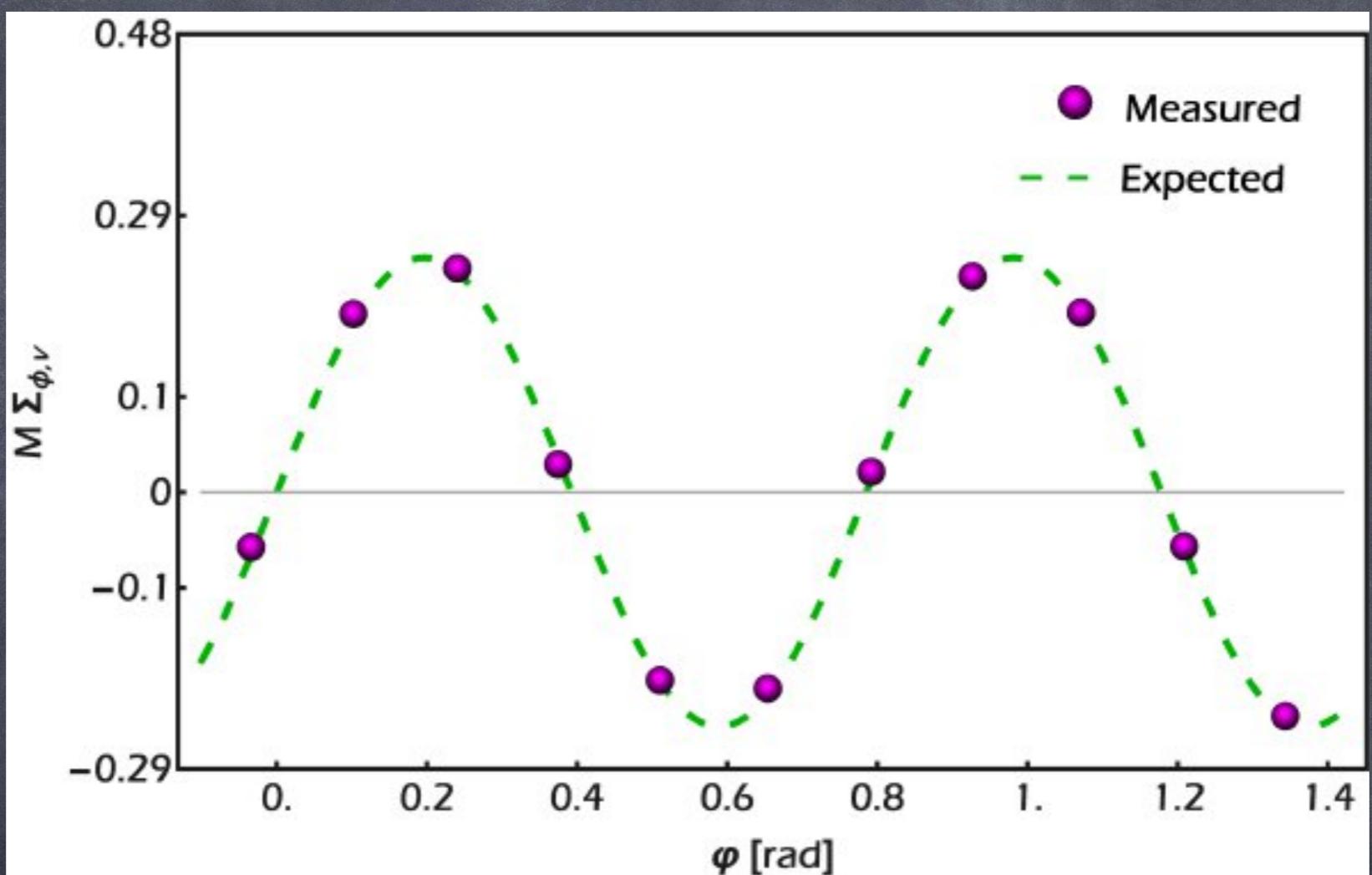


then: are we doing it right?



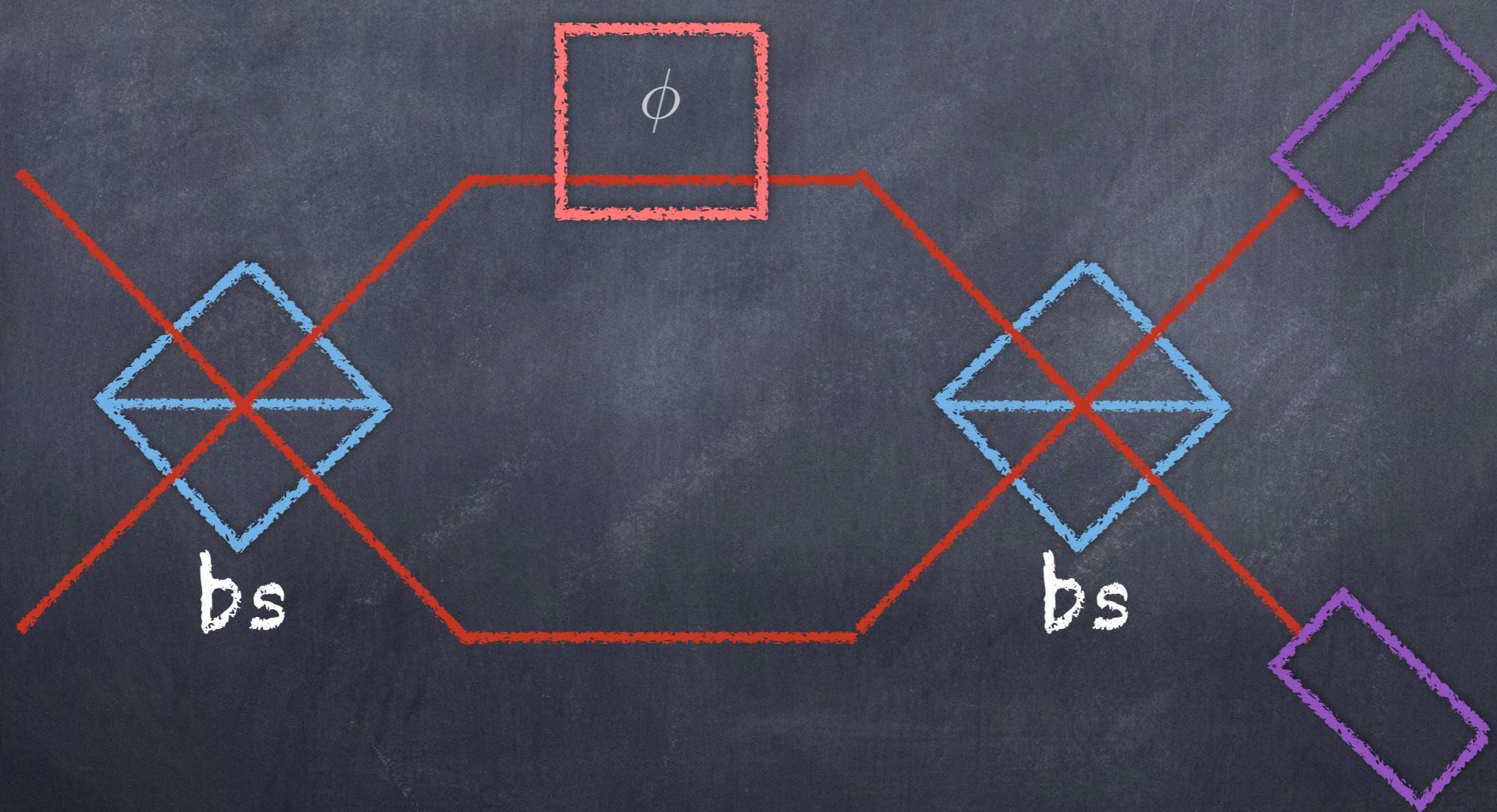
also: are there correlations?

also: are there correlations?



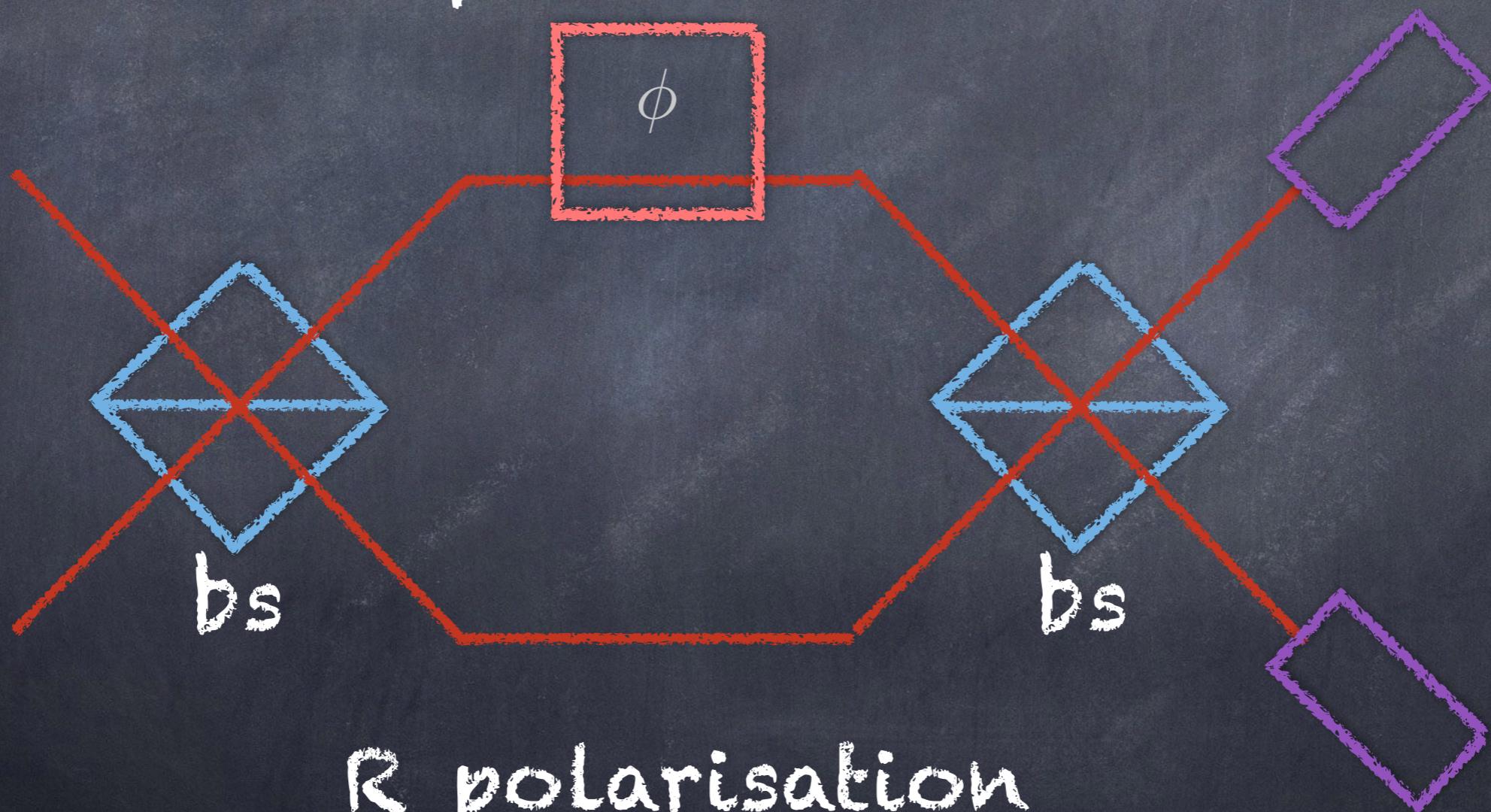
ok, fine: anything interesting?

ok, fine: anything interesting?



ok, fine: anything interesting?

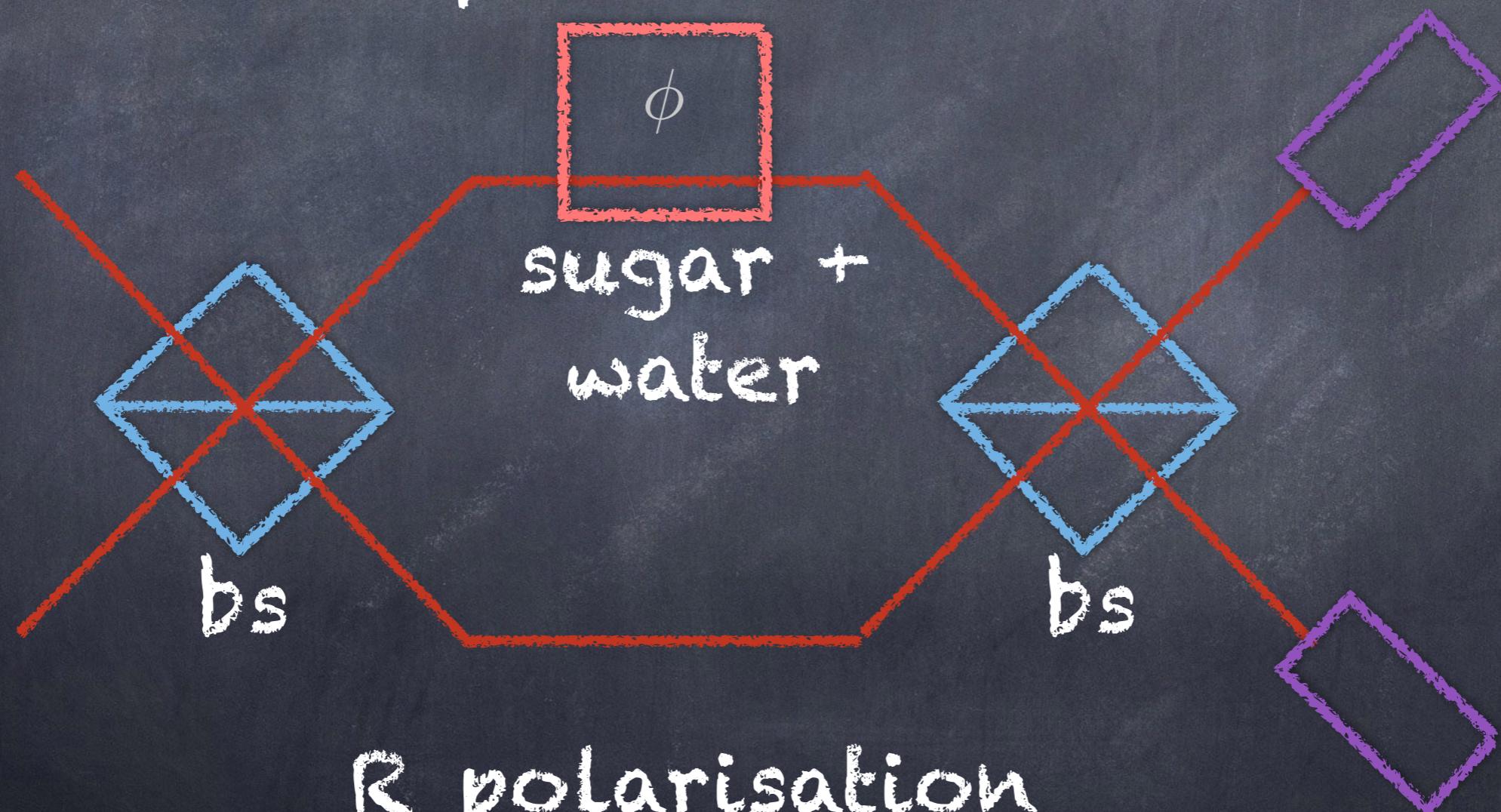
L polarisation



R polarisation

ok, fine: anything interesting?

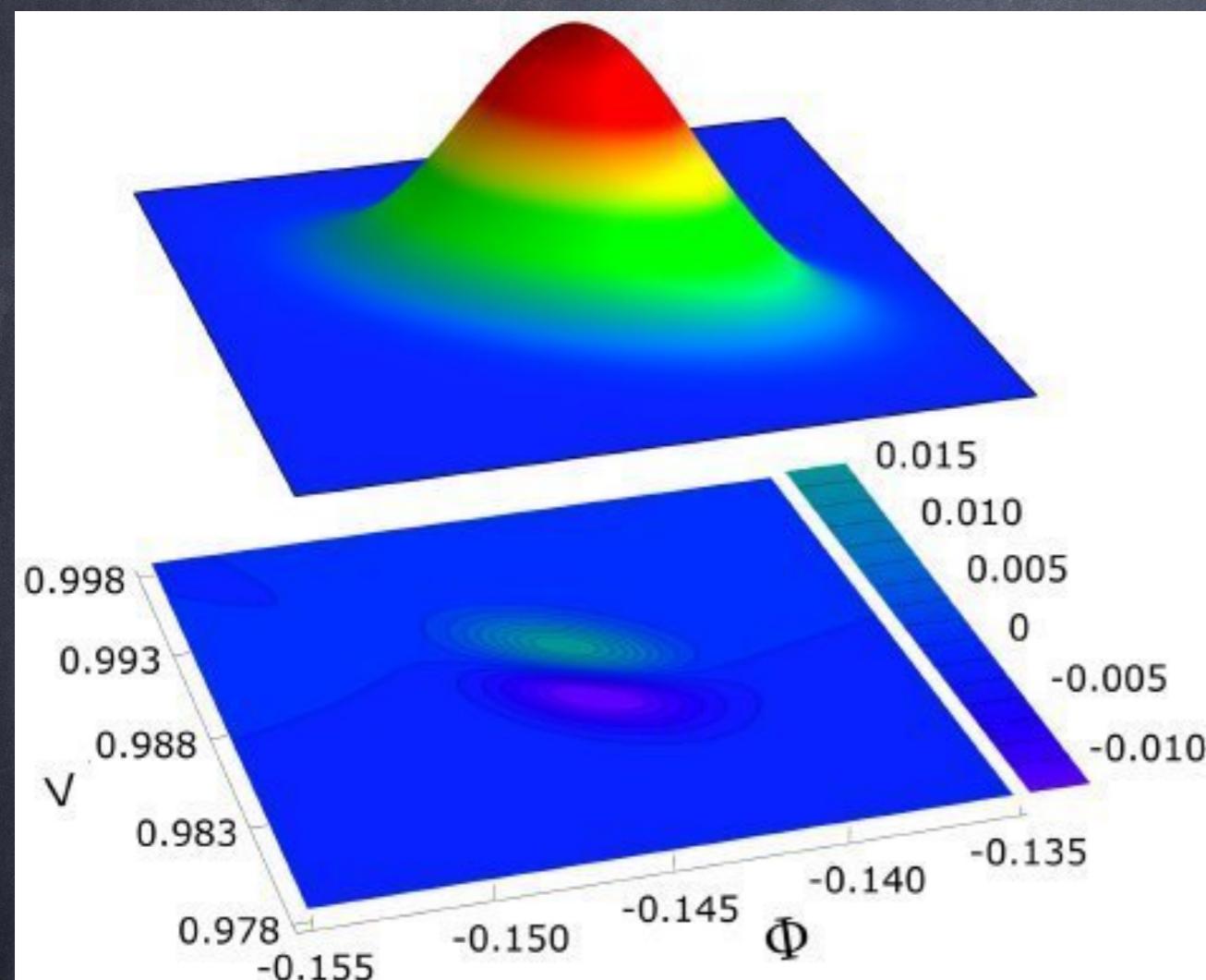
L polarisation



R polarisation

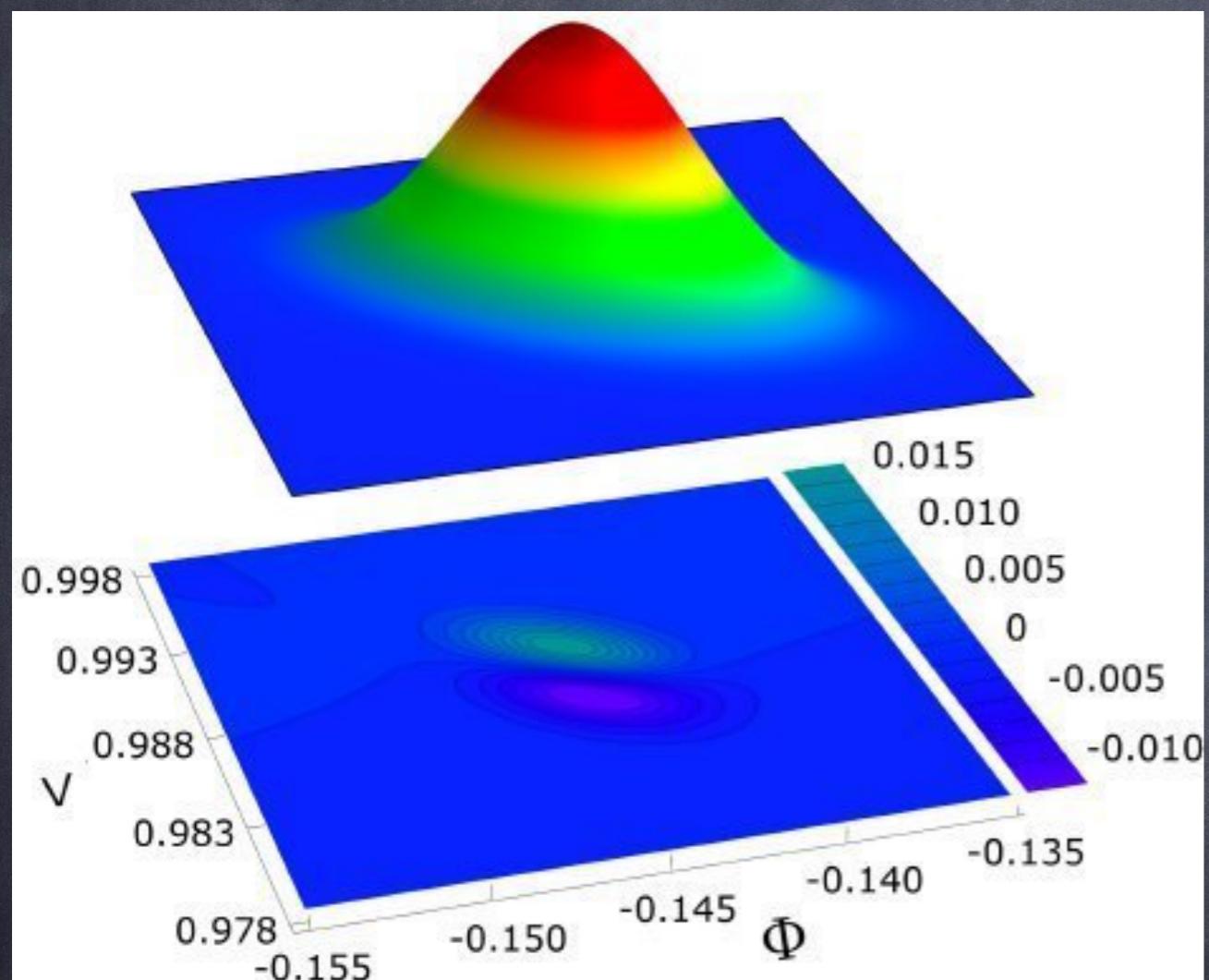
how did it work?

how did it work?

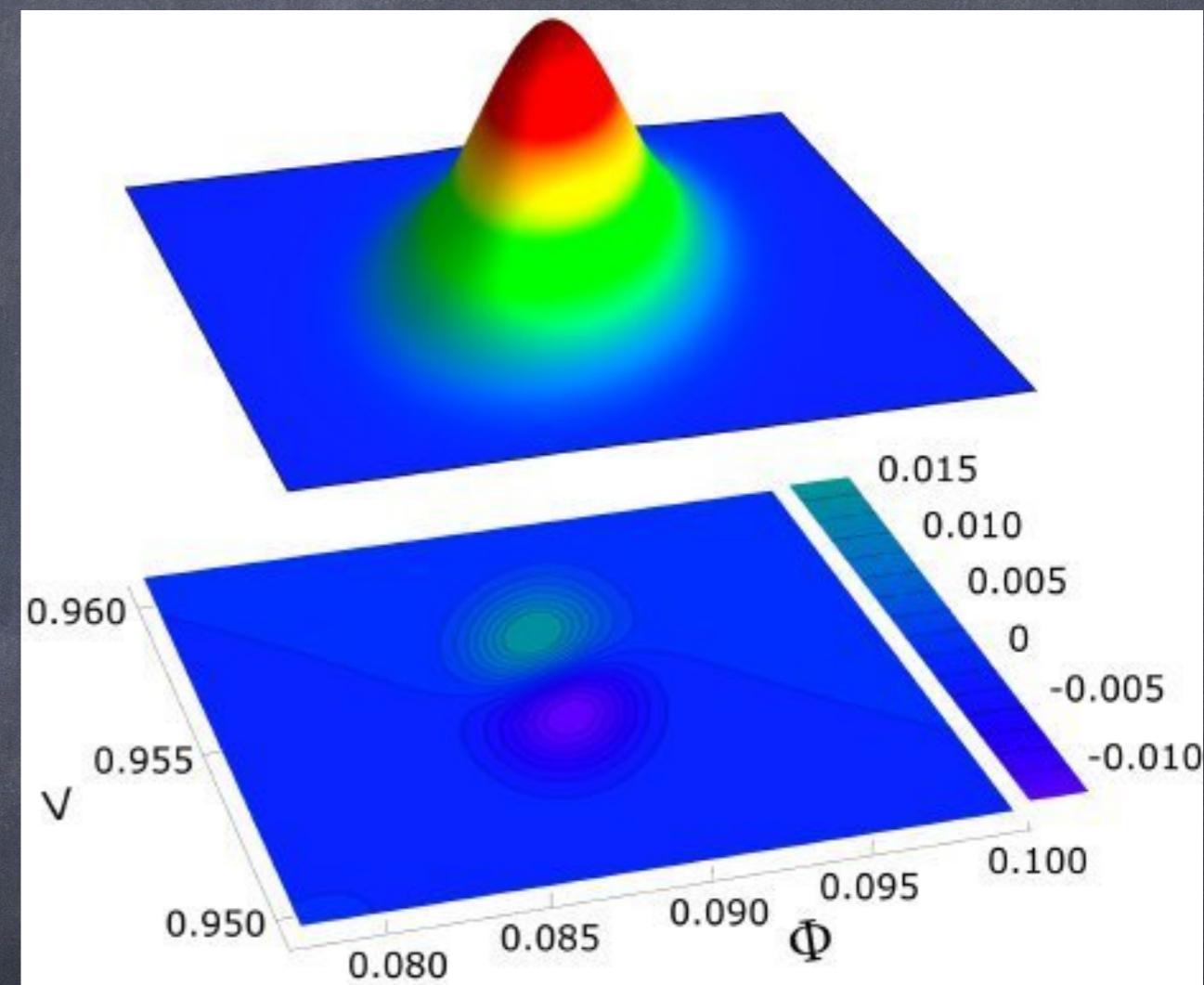


fructose

how did it work?

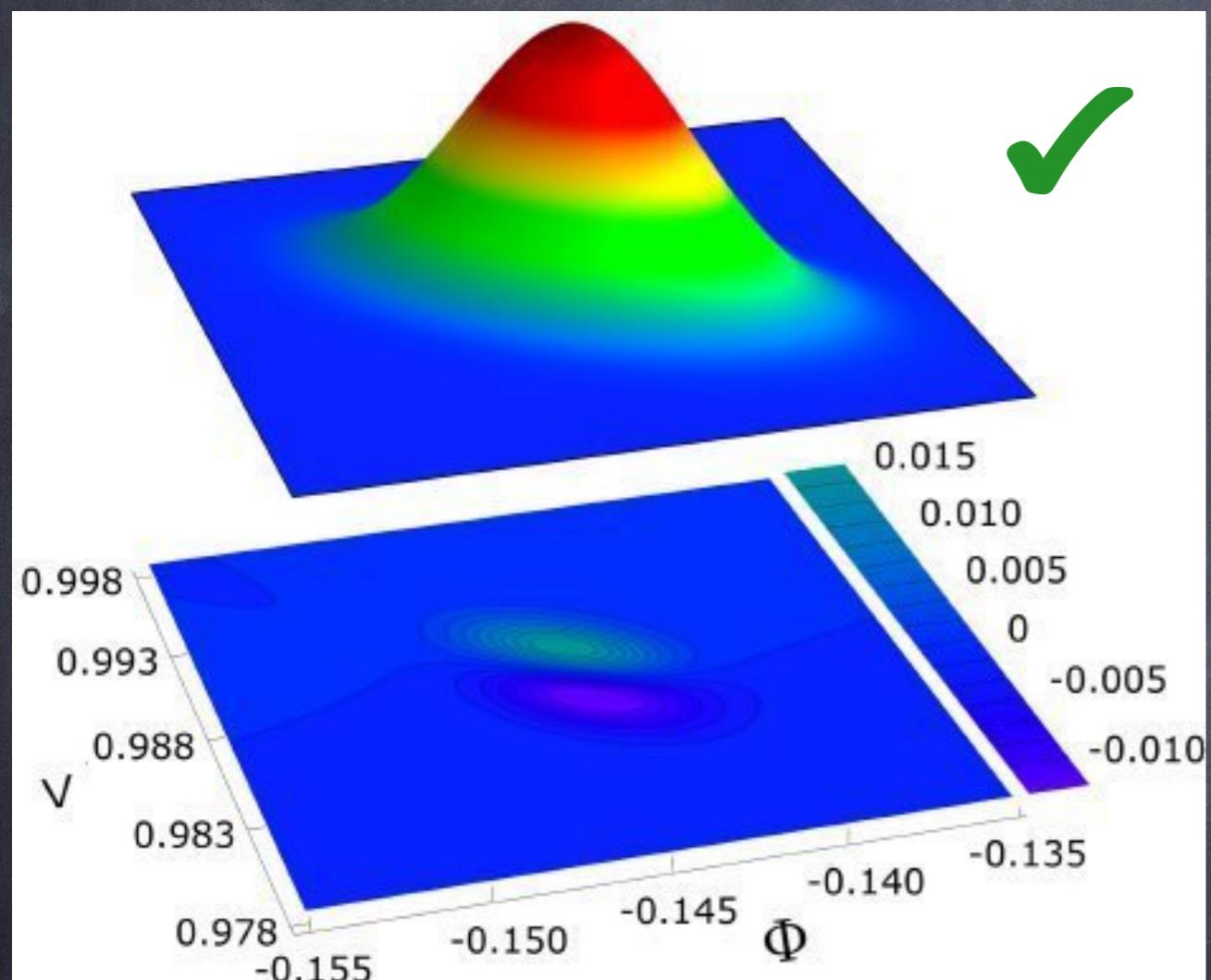


fructose

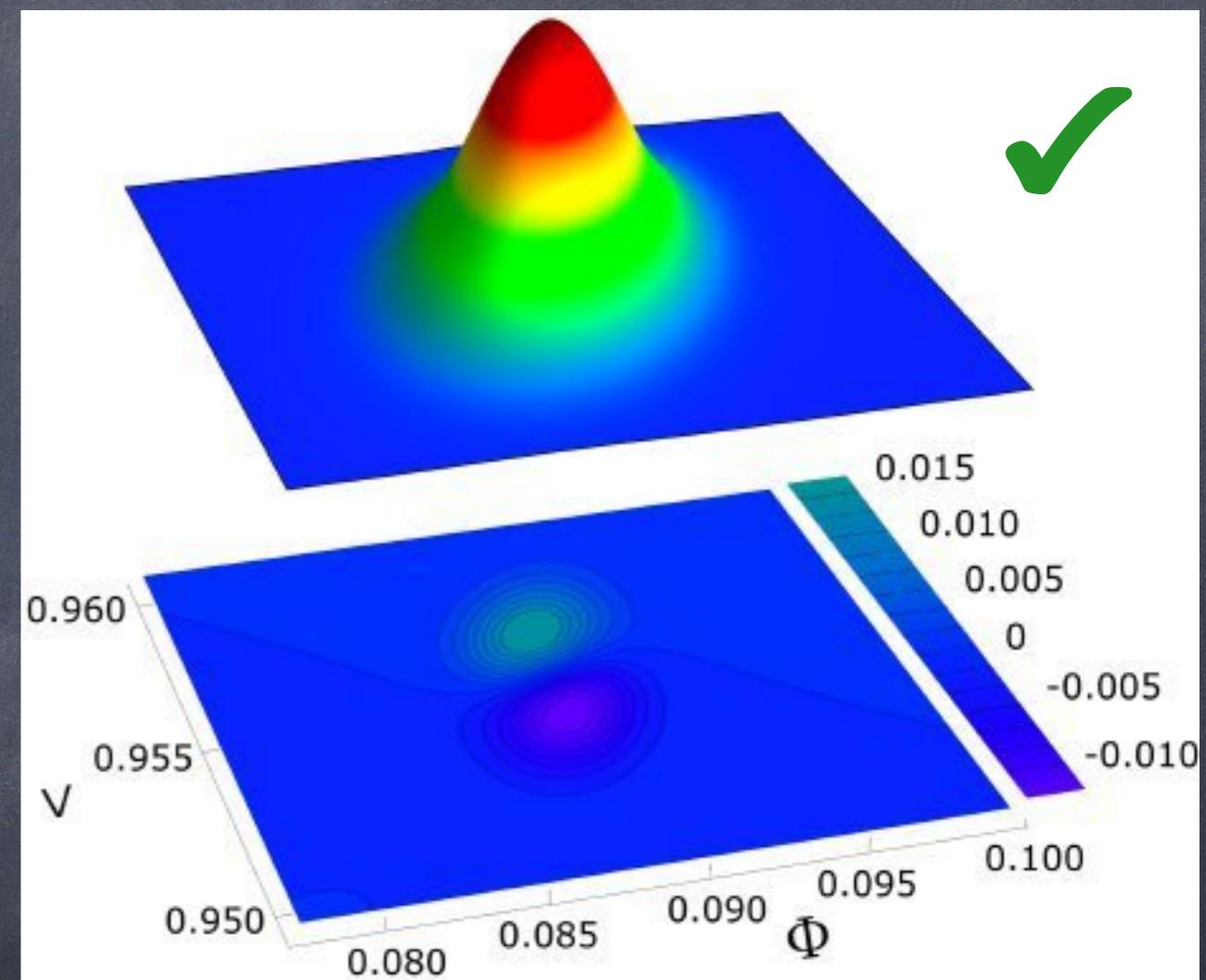


glucose

how did it work?

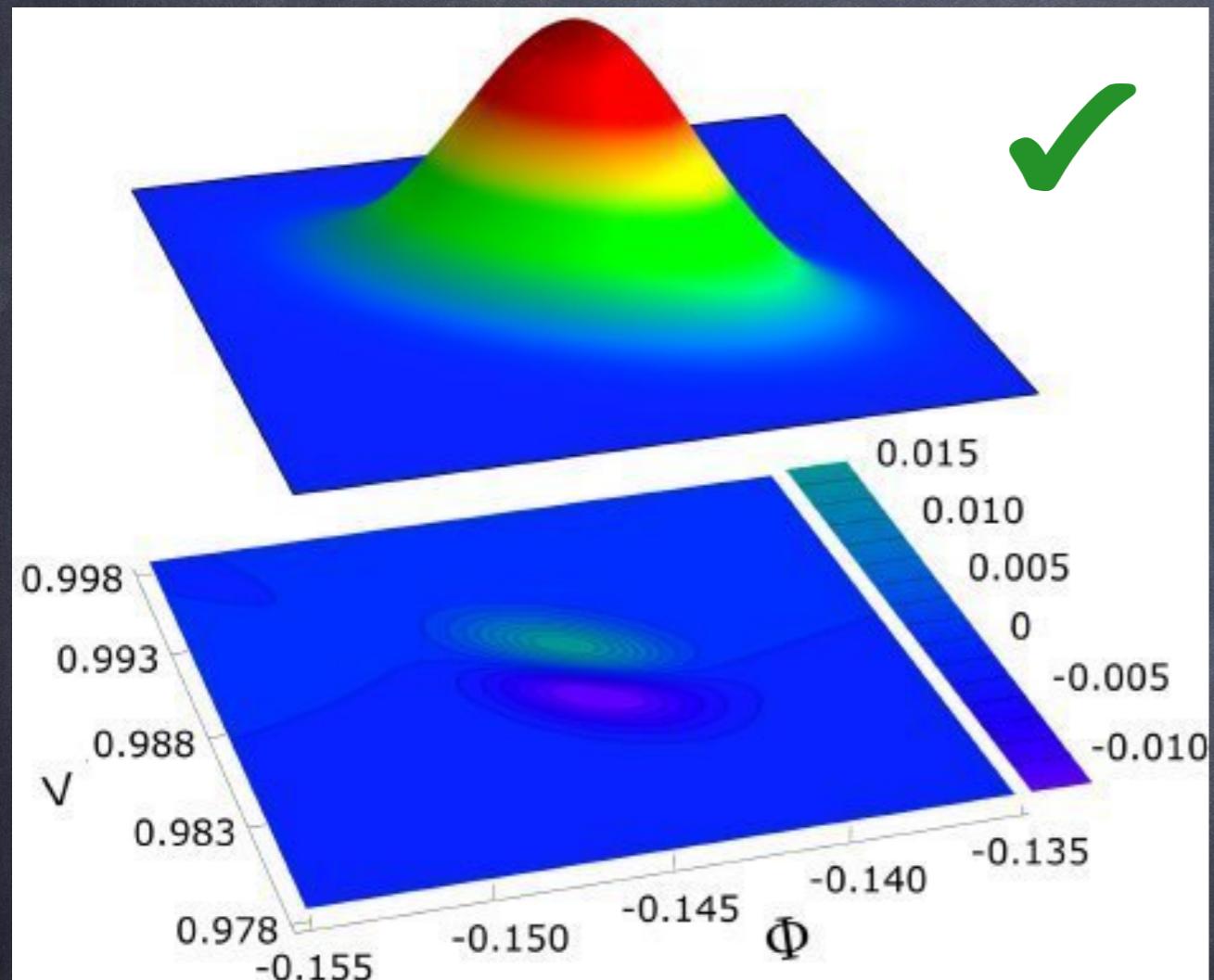


fructose

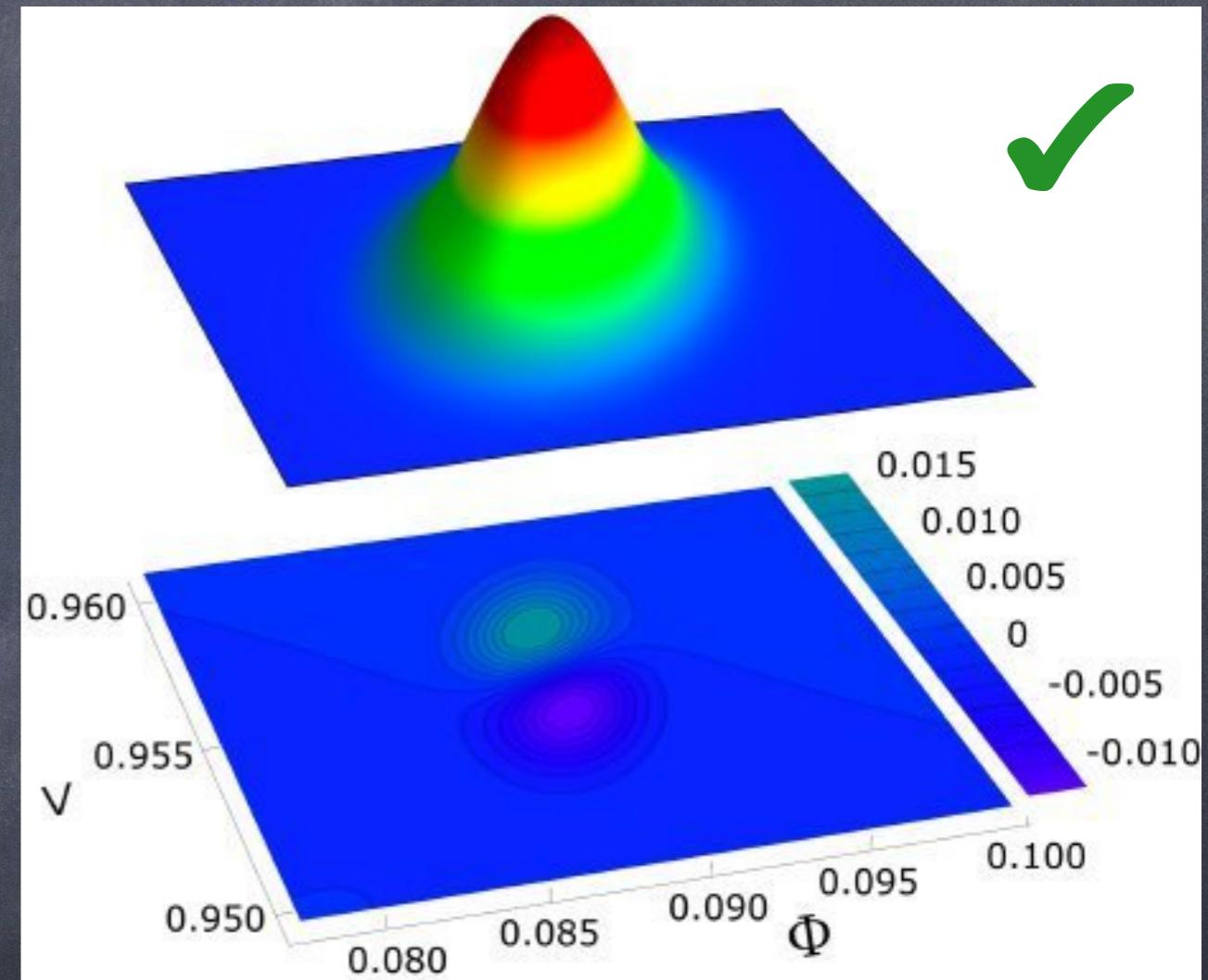


glucose

how did it work?



fructose



glucose

Essential readings

M.G.A. Paris

Quantum estimation for quantum
technologies

Int. J. Quantum Info. 7, 125-137 (2009)

V. Giovannetti, S. Lloyd, L. Maccone

Quantum metrology

Phys. Rev. Lett. 96, 010401 (2006)

Good readings

M Szczypulska, T Baumgratz, A Datta
Multi-parameter quantum metrology
Advances in Physics: X 1 (4), 621–639

S Ragy, M Jarzyna, R Demkowicz-
Dobrzański
Compatibility in multiparameter
quantum metrology
Phys. Rev. A 94 (5), 052108

Good readings

S. Slussarenko, et al.

Unconditional violation of the shot-noise limit in photonic quantum metrology

Nature Photonics 11, 700 (2017)