Modes & States in Quantum Optics

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IUF

modes
in classical and quantum optics

What is a mode ?

mode :

a normalized solution $f_1(r,t)$ of Maxwell equations

$$(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\mathbf{f}_1(\mathbf{r},t) = 0, \quad \nabla \cdot \mathbf{f}_1(\mathbf{r},t) = 0 \tag{1}$$

$$\frac{1}{V}\int_V d^3r\, |\mathbf{f}_1(\mathbf{r},t)|^2 = 1 \tag{2}$$

mode basis :

a complete orthonormal set (f_n) of modes

$$\frac{1}{V}\int_V d^3r\, \mathbf{f}^*_m(\mathbf{r},t) \cdot \mathbf{f}_{m'}(\mathbf{r},t) = \delta_{m\,m'}$$

Description of a classical multimode field

general expression for the complex electric field:

$$\mathbf{E}^+(\mathbf{r},t) = \sum_n E_n\, \mathbf{f}_n(\mathbf{r},t)$$

complex amplitude

$$\mathbf{E}^+(\mathbf{r},t) = \sum_n (E_{n,X} + iE_{n,P})\, \mathbf{f}_n(\mathbf{r},t)$$

"quadrature components"

Separation spatial/temporal modes

z-propagating, multi-transverse, multi-temporal mode field:

$$\mathbf{E}^{(+)}(\mathbf{r},t) = \epsilon_1 \sum_j f_j^{(s)}(\mathbf{r}) \sum_r \mathcal{E}_{j,r} f_r^{(t)}(\tau)$$

$$\tau = t - z/c$$

propagating time

if there is a single transverse mode:

$$E_{long}^{(+)}(\tau) = \sum_r \mathcal{E}_r f_r^{(t)}(\tau)$$

with $$\frac{c}{L}\int d\tau f_r^{(t)}(\tau) f_{r'*}^{t}(\tau) = \delta_{rr'}$$

Description of a quantum multimode field

general expression for the complex electric field operator:

$$\hat{\mathbf{E}}^{+}(\mathbf{r},t) = E_0 \sum_n \hat{a}_n \, \mathbf{f}_n(\mathbf{r},t)$$

$$[\hat{a}_n, \hat{a}^{\dagger}_{n'}] = \delta_{n,n'}$$

$$\hat{\mathbf{E}}^{+}(\mathbf{r},t) = \sum_n (\hat{E}_{n,X} + i\hat{E}_{n,P}) \, \mathbf{f}_n(\mathbf{r},t)$$

QUANTUM OPTICS

$$[\hat{E}_{nX}, \hat{E}_{n'P}] = 2iE_0^2 \delta_{n,n'}$$

the double linearity of quantum optics

$$\hat{\mathbf{E}}^{+}(\mathbf{r},t)=\sum_{n}(\hat{E}_{n,X}+i\hat{E}_{n,P})\,\mathbf{f}_{n}(\mathbf{r},t)$$

-linearity of Quantum Mechanics
-linearity of Maxwell equations

Two Hilbert spaces to consider

- the modal Hilbert space H^{mod}
  of solutions of Maxwell equations
  optical coherence

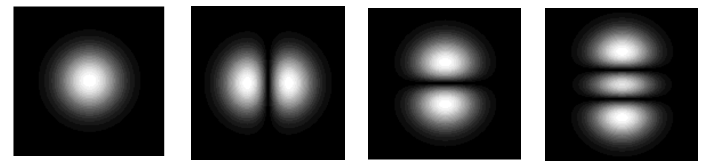
- the quantum Hilbert space H^{q}
  of quantum states of light
  quantum coherence

Different mode bases

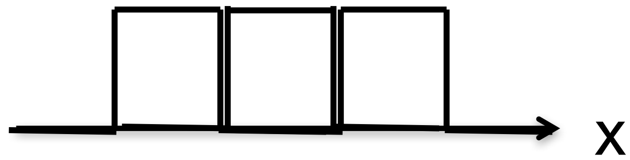
- the travelling plane wave $e^{i(\mathbf{k}.\mathbf{r}-\omega t)}$

transverse/spatial modes

- Hermite Gauss modes

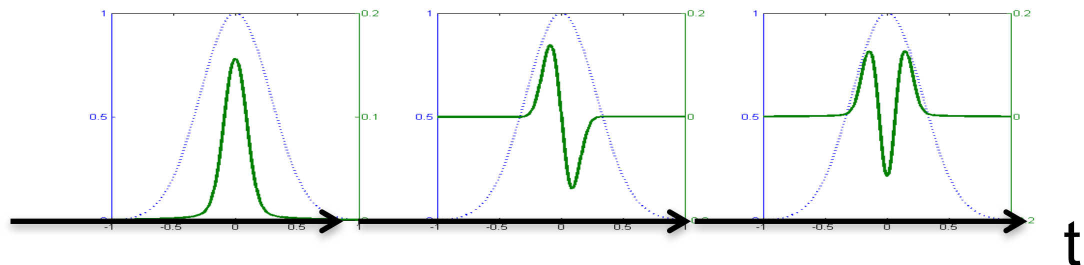


- the pixel modes

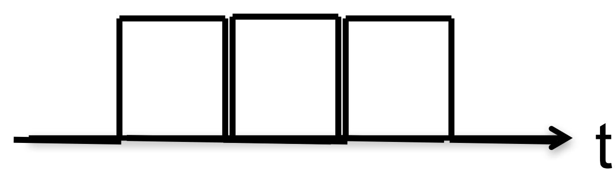


temporal modes

-the temporal
Hermite Gauss pulses



-the time bin modes

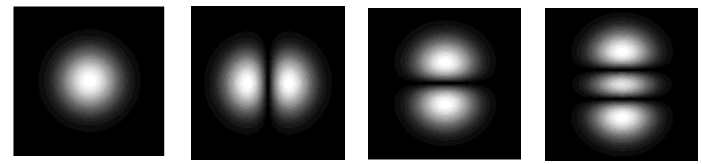


Different mode bases

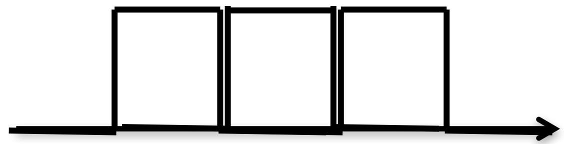
- the travelling plane wave $e^{i(\mathbf{k}.\mathbf{r}-\omega t)}$

transverse/spatial modes

- Hermite Gauss modes



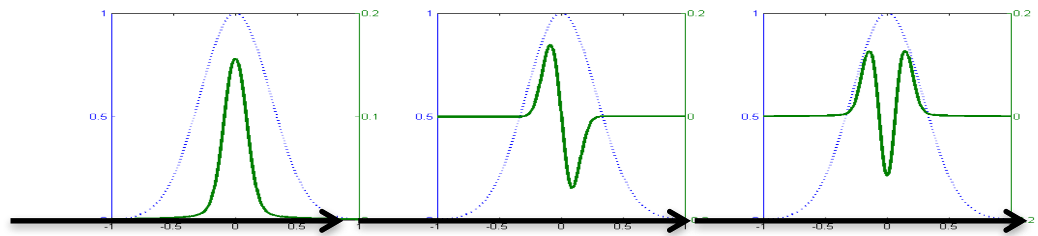
- the pixel modes



x

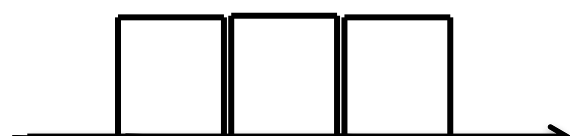
frequency modes

-the multi-frequency Hermite Gauss modes



ω

-the "frequency band" modes
or ‘frexels’



ω

any solution of Maxwell equations is an element of a mode basis

Quantum state in mode basis change

$$\{\mathbf{f}_n\} \longleftrightarrow \{\mathbf{g}_\ell\}$$

$$\mathbf{U}_{mod}$$

$$\{\hat{a}_n\} \longleftrightarrow \{\hat{b}_\ell\}$$

$$|\Psi\rangle = \sum_{p_1}\sum_{p_2} ... A_{p_1,p_2,...}|p_1 : \mathbf{f}_1, p_2 : \mathbf{f}_2, ...\rangle$$

$$= \sum_{q_1}\sum_{q_2} ... B_{q_1,q_2,...}|q_1 : \mathbf{g}_1, q_2 : \mathbf{g}_2, ...\rangle$$

the same quantum state $|\Psi\rangle$ has different expressions in different mode bases

two-mode example:

$$|\Psi\rangle = |squeezed\, vac : \mathbf{f}_1\rangle \otimes |squeezed\, vac : \mathbf{f}_2\rangle$$ $\boxed{\textbf{factorized}}$

$$|\Psi\rangle = |EPR\, entangled\, state\rangle$$ $\boxed{\textbf{entangled}}$

on basis $\mathbf{g}_\pm = (\mathbf{f}_1 \pm \mathbf{f}_2)/\sqrt{2}$

mode basis-independent, or "intrinsic", quantities:

- total photon number:

$$\hat{N}_{tot} = \sum_{\ell} \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell} = \sum_{n} \hat{a}_{n}^{\dagger} \hat{a}_{n}$$

- Wigner function values:

$$W_b(\beta_1, ...) = W_a(\alpha_1, ...)$$

with $$(\beta_1, ...)^T = U(\alpha_1, ...)^T$$

W(0) does not depend on mode basis

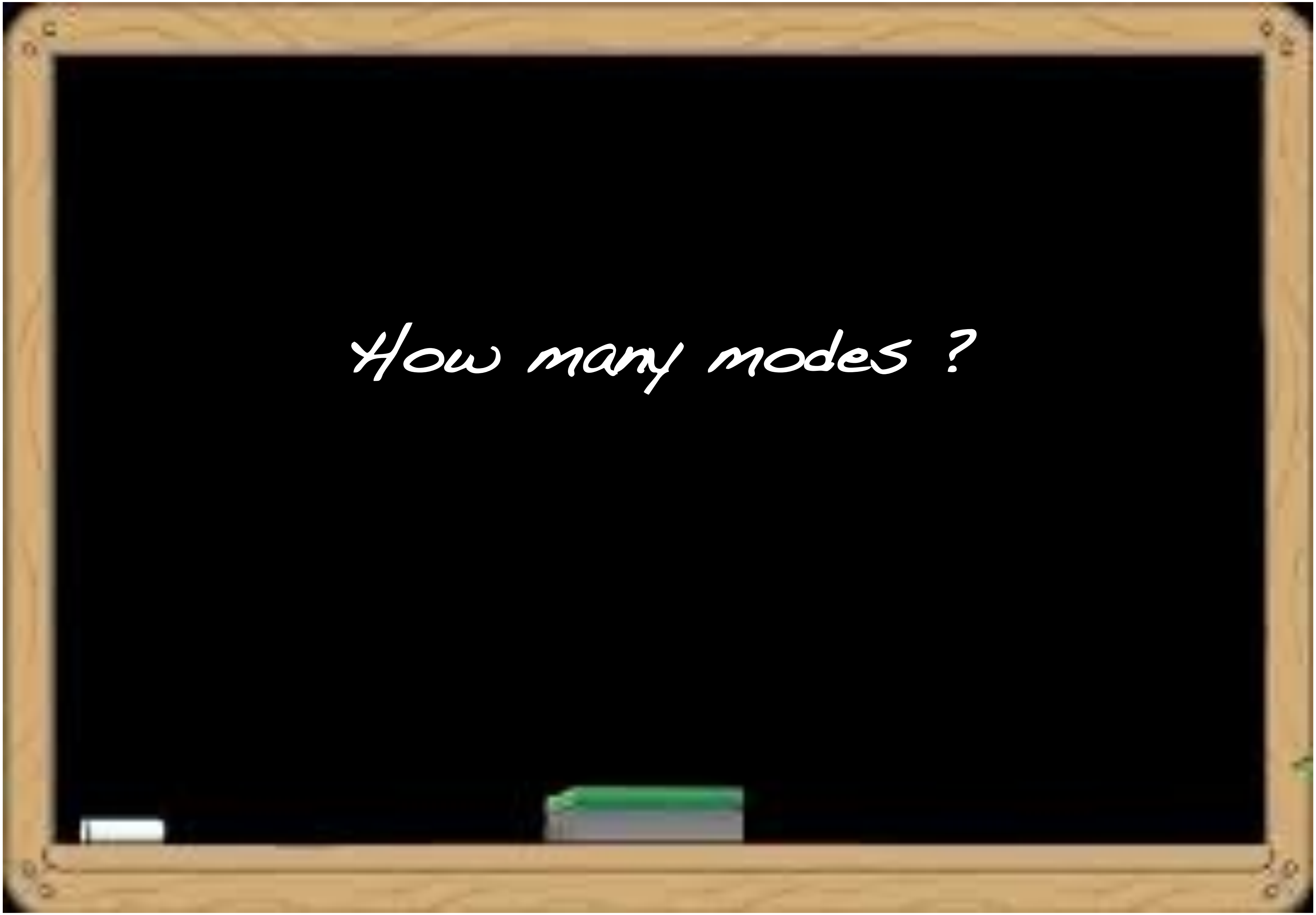
"negativity" of W is intrinsic

hypervolume of negative part is conserved

$$\int d^N \alpha (W(\alpha_1, ...) - |W(\alpha_1, ...)|)$$

- P function values:

"non-classicality " is intrinsic


How many modes ?

counting modes

the number of excited modes is depends on the mode basis

is there a minimum number of modes in which a state lives?

coherence matrix: $C^{i,j}_{coherence} = \langle \hat{a}_i^\dagger \hat{a}_j \rangle$

the coherence matrix can be diagonalized
by a **mode basis change**

counting modes

example: case of three modes, two non-zero eigenvalues

$$C_{complex} = \begin{bmatrix} \boxed{\begin{matrix} N_1 & 0 \\ 0 & N_2 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{bmatrix}$$

$$|\Psi\rangle = \sum_{p_1,p_2} A_{p_1,p_2} |p_1 : \mathbf{g}_1, p_2 : \mathbf{g}_2, 0 : \mathbf{g}_3\rangle$$ intrinsic two-mode state

the number of non-zero eigenvalues (rank) of covariance matrix is the minimum number of modes needed to describe the quantum state

the corresponding Hilbert space is the smallest space in which the state is living

useful for example to make full tomography of the state

intrinsic **single mode** state

its coherence matrix has only one nonzero eigenvalue

there is a mode basis $\{\mathbf{g}_\ell\}$
in which it is single mode

$$|\Psi\rangle = \left(\sum A_{q_1} |q_1 : \mathbf{g}_1\rangle\right) \otimes |0 : \mathbf{g}_2\rangle \otimes |0 : \mathbf{g}_3\rangle \otimes \ldots$$

the **single photon** state: an intrinsic single mode state

defined as eigenstate of $\hat{N}_{tot}$ with eigenvalue 1

$$|\Psi_1\rangle = \sum_n A_n |1 : \mathbf{f}_n\rangle$$

can be written: $|\Psi_1\rangle = |1 : \mathbf{g}_1\rangle \otimes |0, 0, ...\rangle$

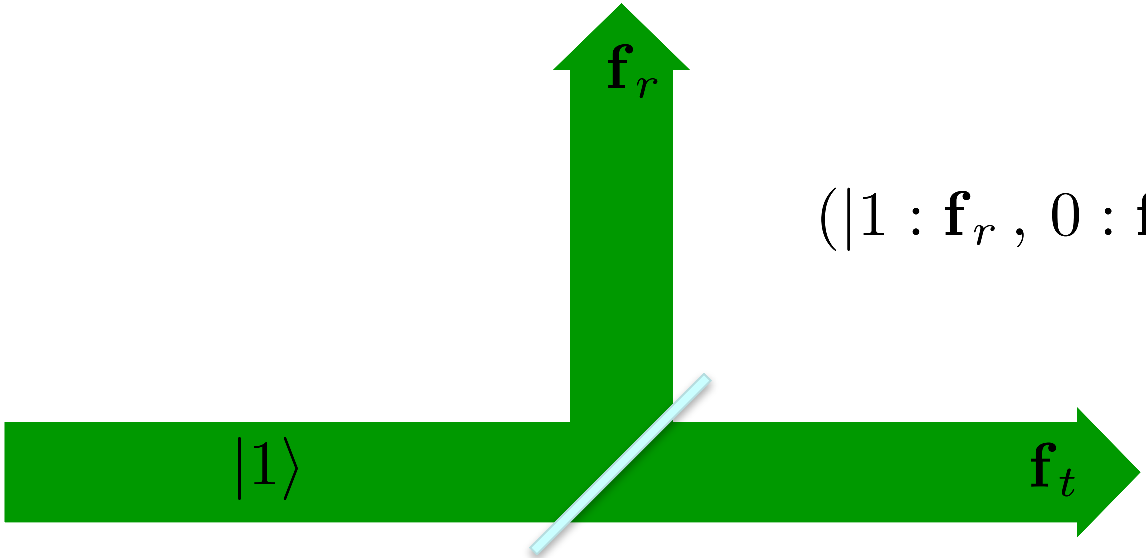
with $\mathbf{g}_1 = \sum_n A_n \mathbf{f}_n$

a single photon state is always a single mode state

a single photon state cannot be defined independently of the mode in which it is defined

its properties depend on this mode

example: single photon through beamsplitter



$$(|1 : \mathbf{f}_r\ ,\ 0 : \mathbf{f}_t\rangle + |0 : \mathbf{f}_r\ ,\ 1 : \mathbf{f}_t\rangle)/\sqrt{2}$$

single photon in single mode $\mathbf{g}_1 = (\mathbf{f}_r + \mathbf{f}_t)/\sqrt{2}$

single mode states and optical coherence

in any single mode state :

$$|g^{(1)}| = 1$$

with

$$g^{(1)} = \frac{< E^{(-)}(\mathbf{r},t)E^{(+)}(\mathbf{r}',t') >}{(< E^{(-)}(\mathbf{r},t)E^{(+)}(\mathbf{r},t) >< E^{(-)}(\mathbf{r}',t')E^{(+)}(\mathbf{r}',t') >)^{1/2}}$$

interferences are of perfect visibility
whatever the quantum state

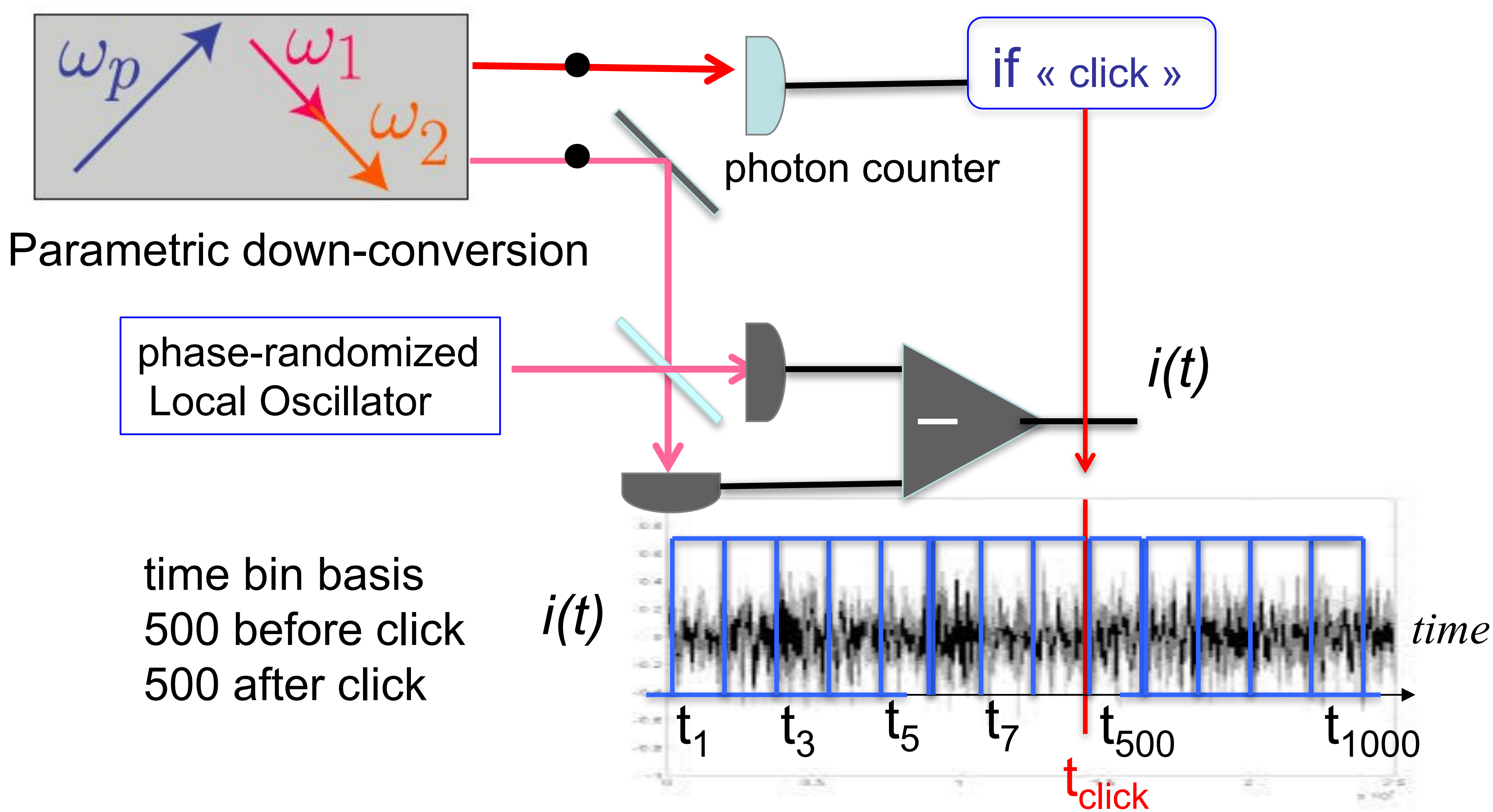
interference visibility: a mode property, not a state property

(R. Glauber)

Determination of the mode of a single photon state

Determination of the mode of a heralded single photon

O. Morin, C. Fabre, J. Laurat PRL **111**, 213602 (2013)



related work : A. Mc Rae,T. Brannan , R. Rachal, A. Lvovsky PRL **109**, 033601 (2012)

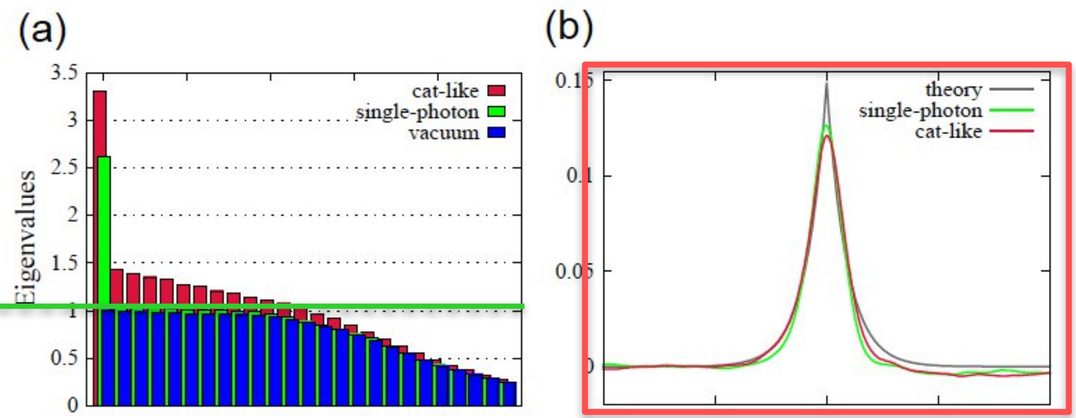
averaged over
a great number of clicks

one gets a 1000×1000 matrix :

$$C^{exp}_{n,n'} = \overline{i(t_n)i(t_{n'})}$$
$$= 2\,C_{coherence} + 1$$

diagonalisation of C^{exp} :

vacuum level



eigenvalues

eigenstate of non-zero eigenvalue

only one eigenvalue different from vacuum fluctuations

the generated state is indeed single mode

the corresponding eigenstate gives the
shape of the temporal mode of the single photon

Complete characterization of multimode Gaussian states

The covariance matrix

the *coherence* matrix $C^{i,j}_{coherence} = \langle \hat{a}_i^\dagger \hat{a}_j \rangle$
can be used to count modes on any quantum state, Gaussian or not, but it gives partial information

in contrast Gaussian states are completely characterized by the covariance matrix

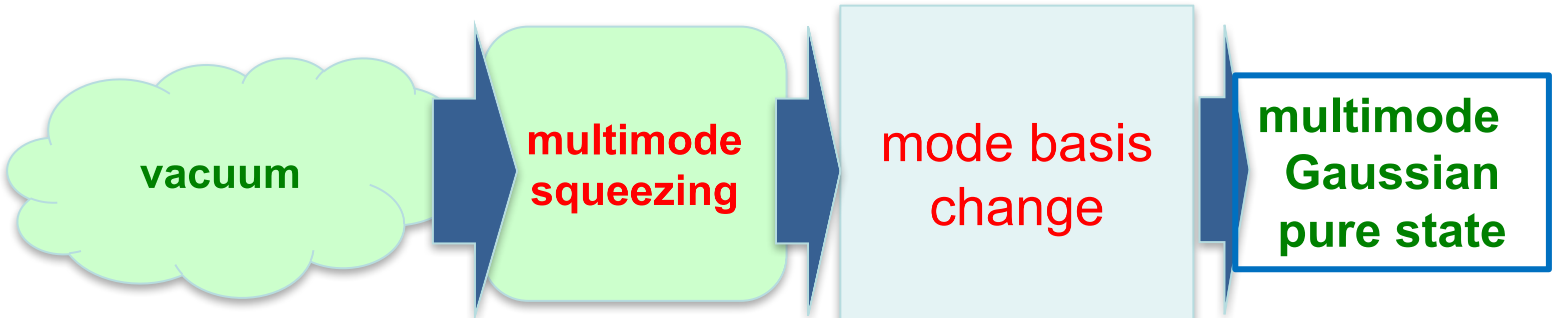
$$\begin{bmatrix} \Delta^2 E_{X1} & < E_{X1} E_{X2} > & < E_{X1} E_{P1} > & < E_{X1} E_{P2} > \\ < E_{X2} E_{X1} > & \Delta^2 E_{X2} & < E_{X2} E_{P1} > & < E_{X2} E_{P2} > \\ < E_{P1} E_{X1} > & < E_{P1} E_{X2} > & \Delta^2 E_{P1} & < E_{P1} E_{P2} > \\ < E_{P2} E_{X1} > & < E_{P2} E_{X2} > & < E_{P2} E_{P1} > & \Delta^2 E_{P2} \end{bmatrix}$$

of dimension $(2N_{modes}) \times (2N_{modes})$

it can be diagonalized, but the corresponding linear transformation is not a mode basis change

characterization
of a multimode Gaussian quantum state:

for a pure state:



uncorrelated squeezed mode basis	turns squeezed mixed states into entangled states
$\{\mathbf{g}_\ell\}$	$\mathbf{U}^2_{mod}$

Bloch Messiah (or Singular Value) decomposition

characterization of a multimode Gaussian quantum state:

for a mixed state:

uncorrelated thermal states → mode basis change → **multimode squeezing** → mode basis change → **multimode Gaussian mixed state**

uncorrelated thermal mode basis	produces correlated classical noise	uncorrelated squeezed mode basis	turns squeezed mixed states into entangled states
$\{\mathbf{f}_n\}$	$\mathbf{U}^1_{mod}$	$\{\mathbf{g}_\ell\}$	$\mathbf{U}^2_{mod}$

two mode basis

Bloch Messiah Williamson reduction

characterization of a multimode Gaussian quantum state:

- all pure multimode Gaussian states are **factorizable**
  there is a mode basis in which:

$$W(x_1, p_1, ...x_n, p_n) = W_1(x_1, p_1) \times ... \times W_n(x_n, p_n)$$

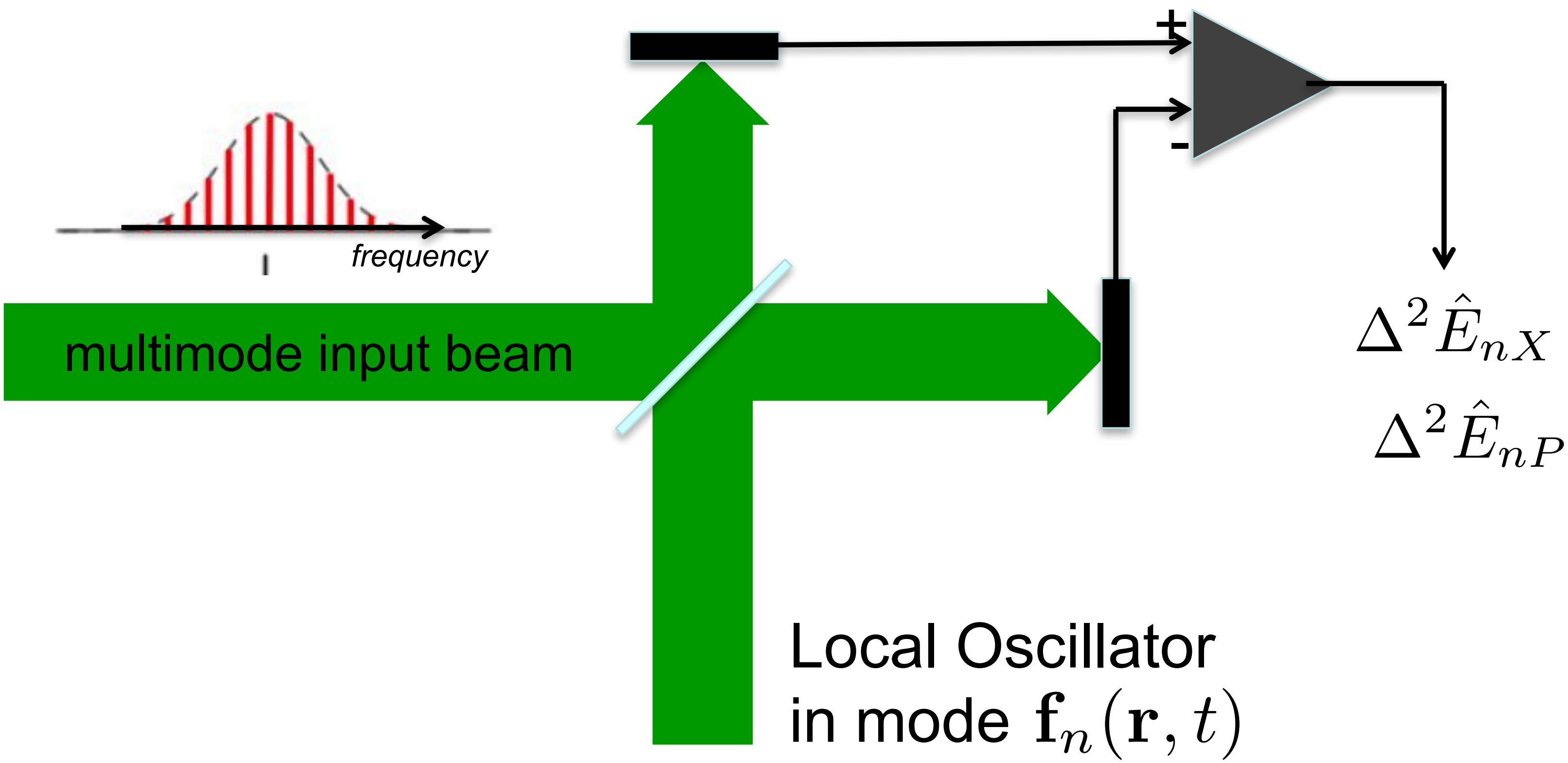
- all mixed multimode Gaussian states are **separable**
  there is a mode basis in which

$$W(x_1, p_1, ...x_n, p_n) = \int d\lambda p(\lambda) W_{1,\lambda}(x_1, p_1) \times ... \times W_{n,\lambda}(x_n, p_n)$$

there are no intrinsically entangled Gaussian states

How to measure the covariance matrix?

balanced homodyne detection gives information about the **projection** of the multimode state on the local oscillator mode

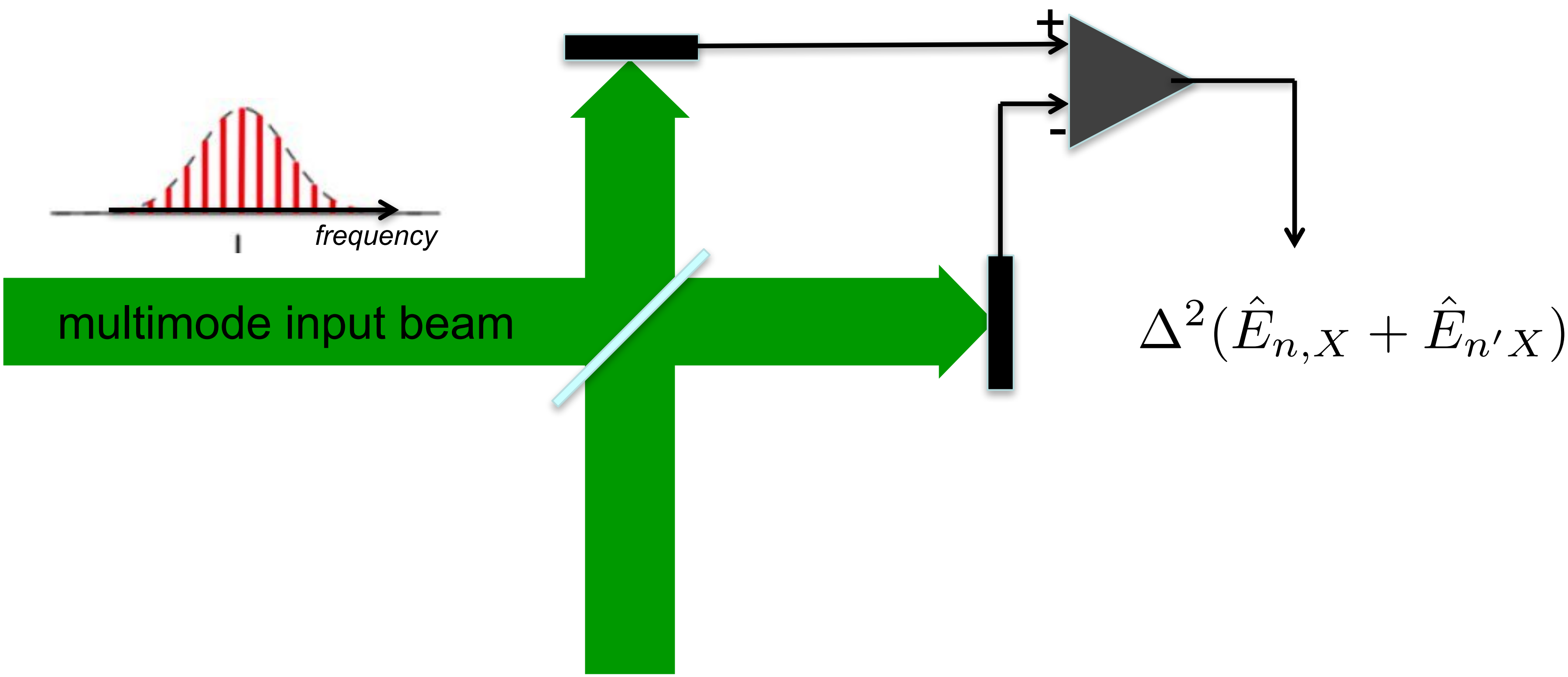


make a series of homodyne measurements using a set of orthogonal modes $\mathbf{f}_n(\mathbf{r}, t)$

How to measure the off-diagonal part of the covariance matrix?

make homodyne measurements
using the sum of two modes

$$\langle \hat{E}_{nX} \hat{E}_{n'X} \rangle = (\Delta^2(\hat{E}_{nX} + \hat{E}_{n'X}) - \Delta^2 \hat{E}_{nX} - \Delta^2 \hat{E}_{n'X})/2$$

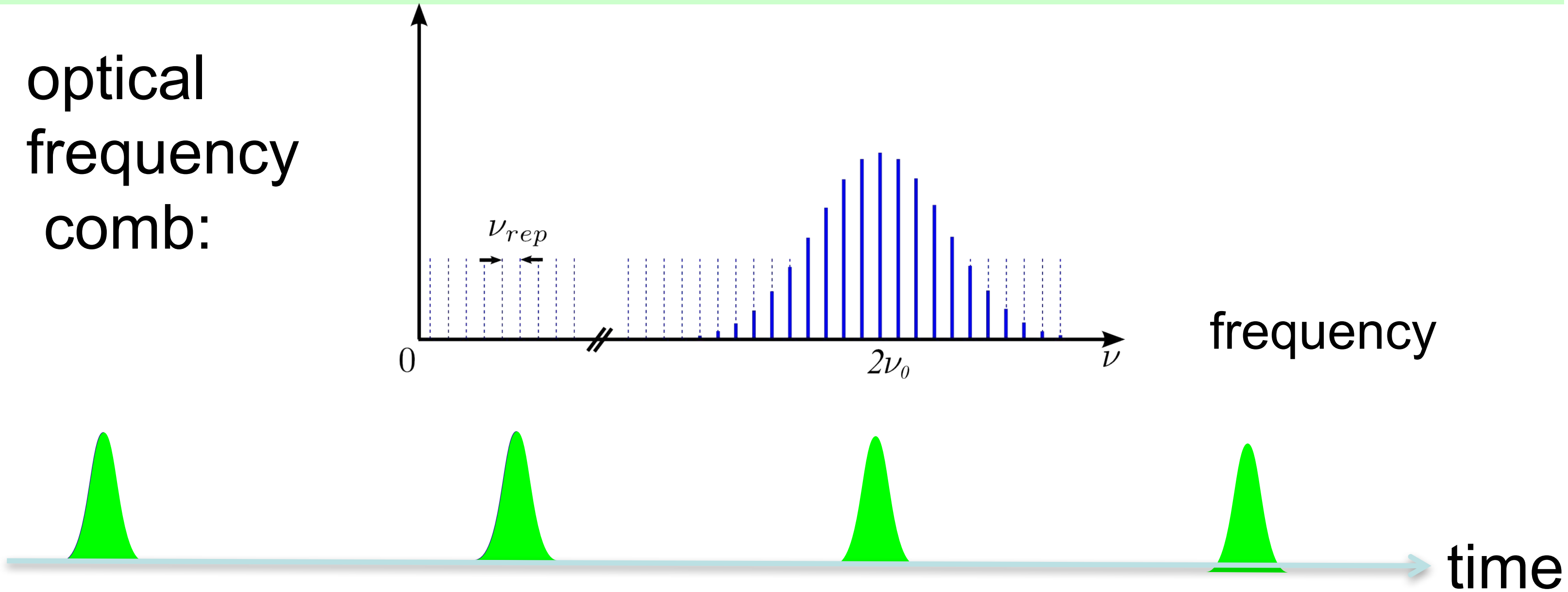


Local Oscillator
in mode $\mathbf{f}_n + \mathbf{f}_{n'}$

Generation and characterization of a highly multimode Gaussian non-classical state

A multi-frequency-mode light:
the frequency comb generated by a mode-locked laser

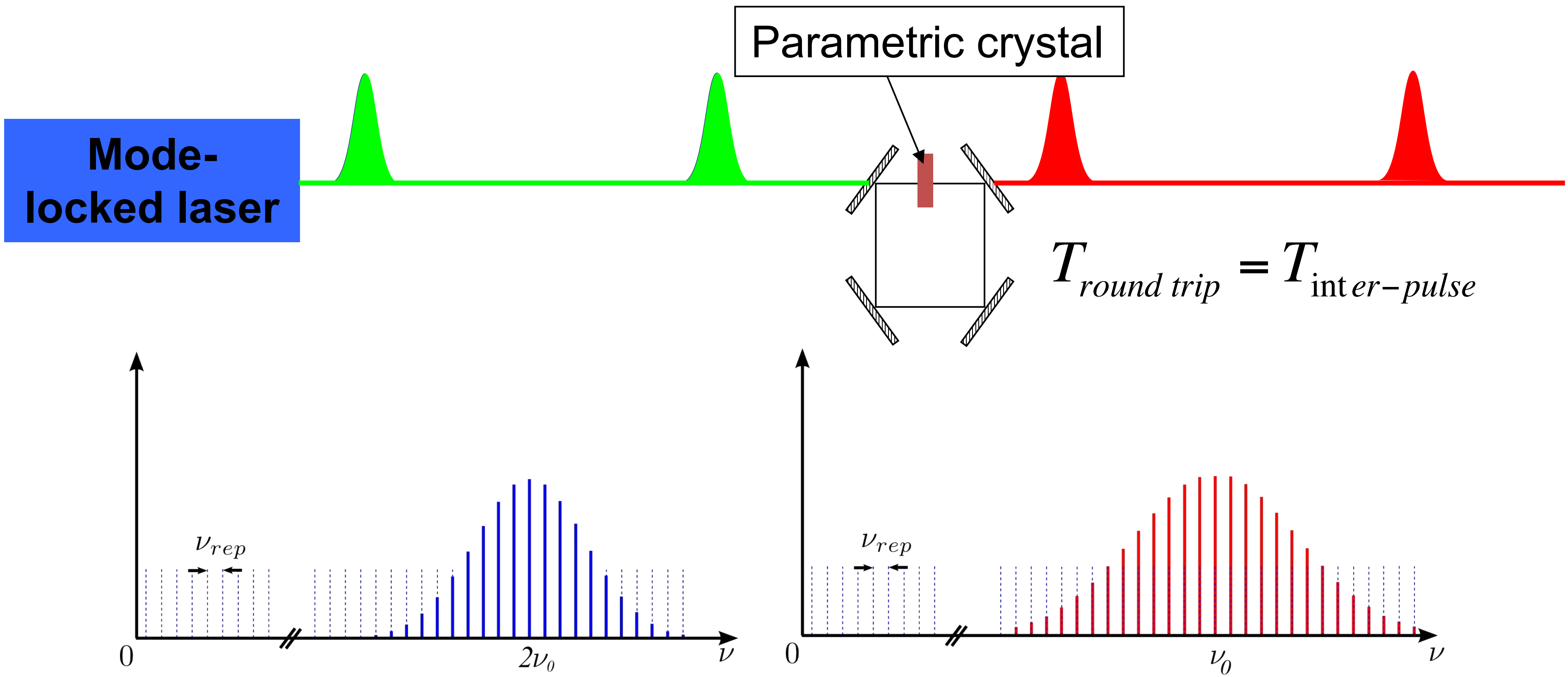
optical
frequency
comb:



Frequency modes of a mode-locked laser: **about 100.000**

a quantum frequency comb ?

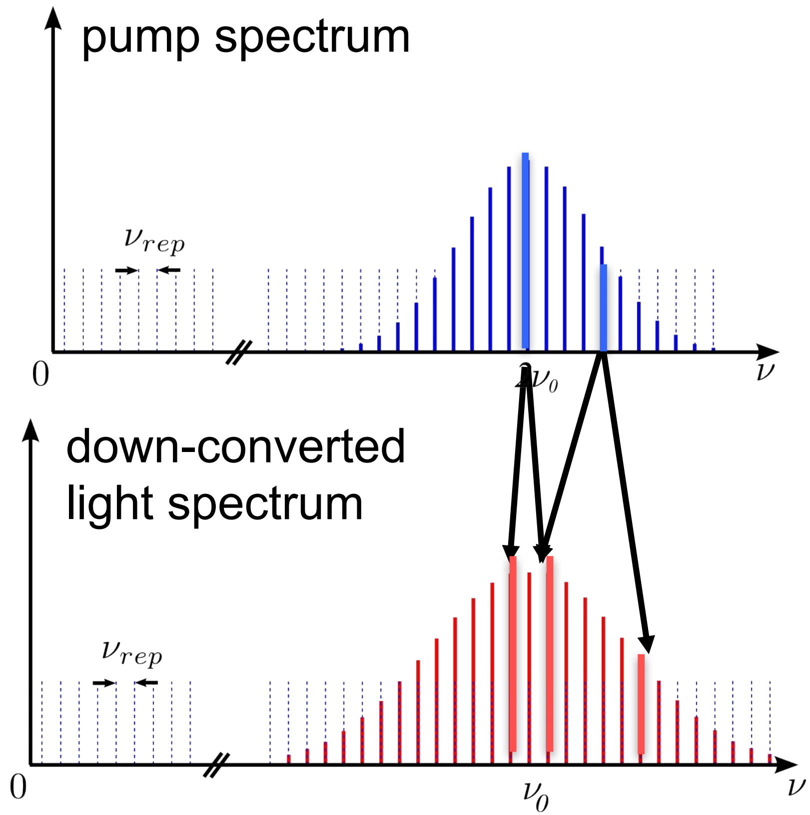
generation by parametric down conversion
of a mode-locked laser



the 10^5 frequency modes : are they entangled ?

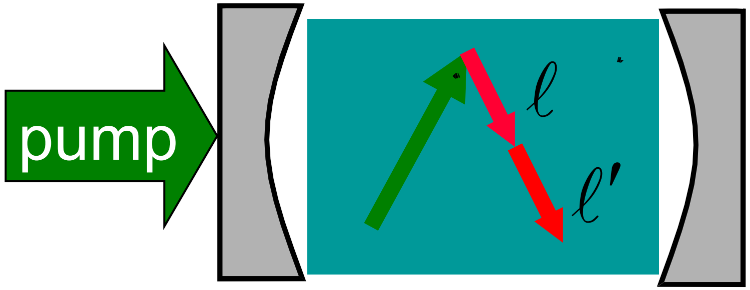
Generation of a multimode quantum state from multimode pump

parametric down conversion of a monochromatic pump gives rise to EPR entangled signal and idler beams



all couples of frequencies modes should be entangled !

A liitle bit of theory ...



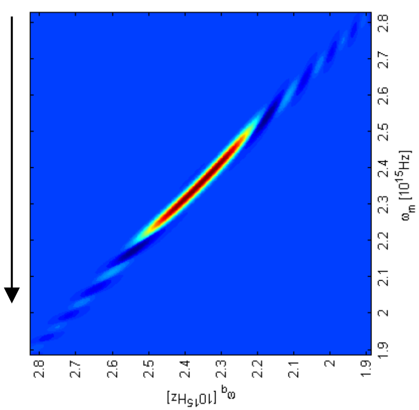
$$\hat{H} = \sum_{\ell,\ell'} G_{\ell,\ell'} \left(\hat{a}_{\ell}^{+} \hat{a}_{\ell'}^{+} + \hat{a}_{\ell} \hat{a}_{\ell'} \right)$$

Symmetrical matrix

ℓ'

ℓ

$G_{\ell,\ell'}$



G. De Valcarcel, G. Patera, N. Treps, C. Fabre, Phys. Rev. A**74,** 061801(R) (2006)

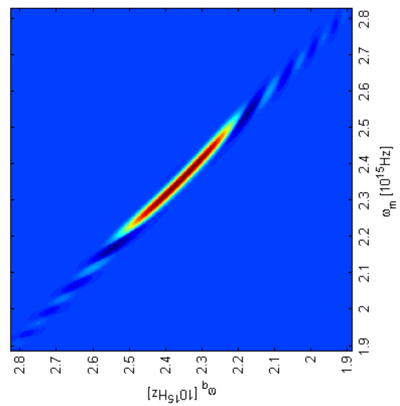
Shifeng Jiang, N. Treps, C. Fabre, New Journal of Physics, **14** 043006 (2012)

Diagonalizing the interaction

ℓ'

$G_{\ell,\ell'}$

ℓ



- Eigenstates:

linear combinations of frequency modes
« supermodes »

$$\hat{b}_k = \sum_{\ell} U_k^{\ell} \hat{a}_\ell$$

- eigenvalues

$$\Lambda_k$$

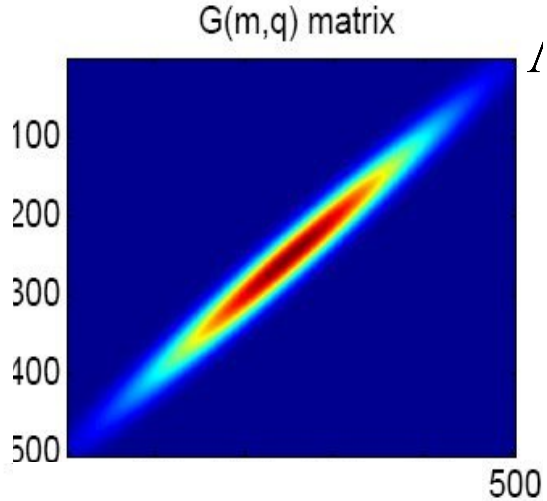
: multi-squeezing hamiltonian

$$\hat{H} = \hbar \sum_{k=1}^{N_m} \Lambda_k \left(\hat{b}_k^2 + \hat{b}_k^{+2} \right)$$

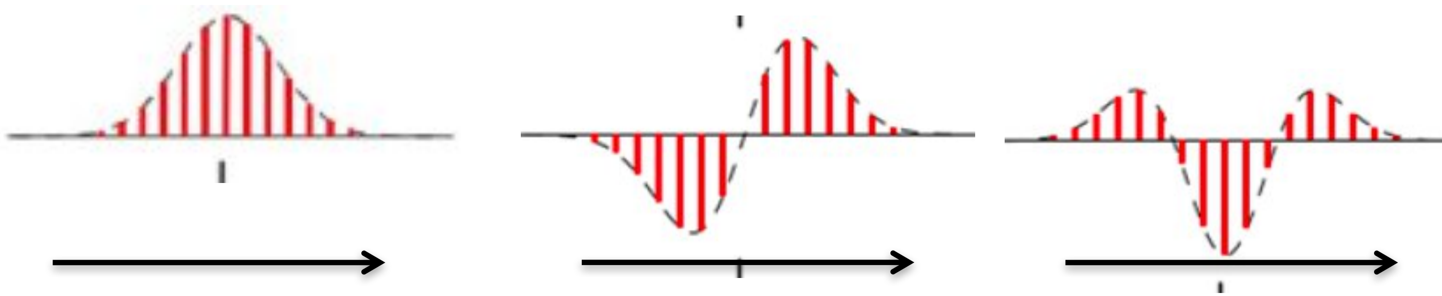
$$\left|\Psi_{\text{out}}\right\rangle = \left|Squeezed\ state_k(\Lambda_1)\right\rangle \otimes \ldots \otimes \left|Squeezed\ state_k(\Lambda_{N_m})\right\rangle \otimes \left|0\right\rangle \otimes \ldots$$

supermode shapes

Simple example: Gaussian variation of $G_{\ell,\ell'}$



Eigenmodes: combs with Hermite-Gauss modal amplitudes

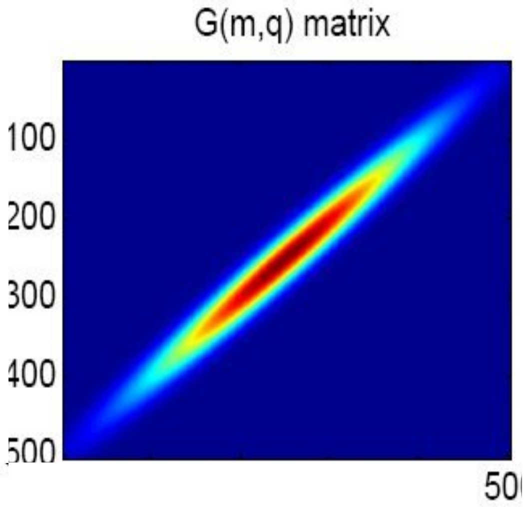


frequency

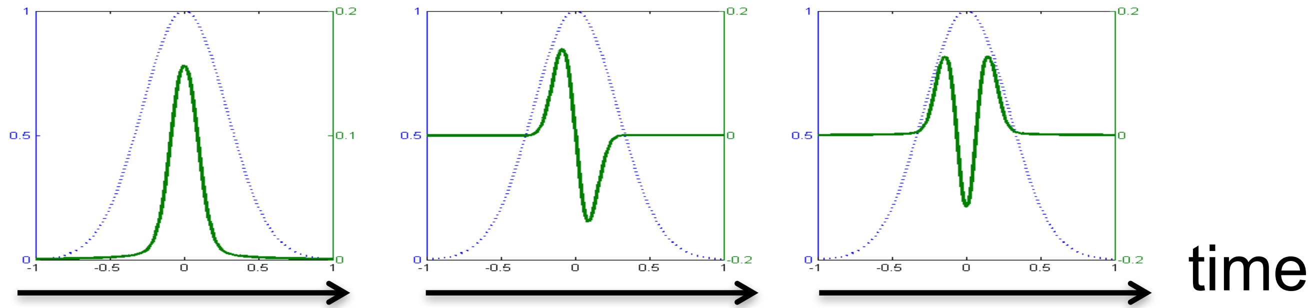
HG_0 HG_1 HG_2

supermode shapes

Simple example: Gaussian variation of $G_{\ell,\ell'}$

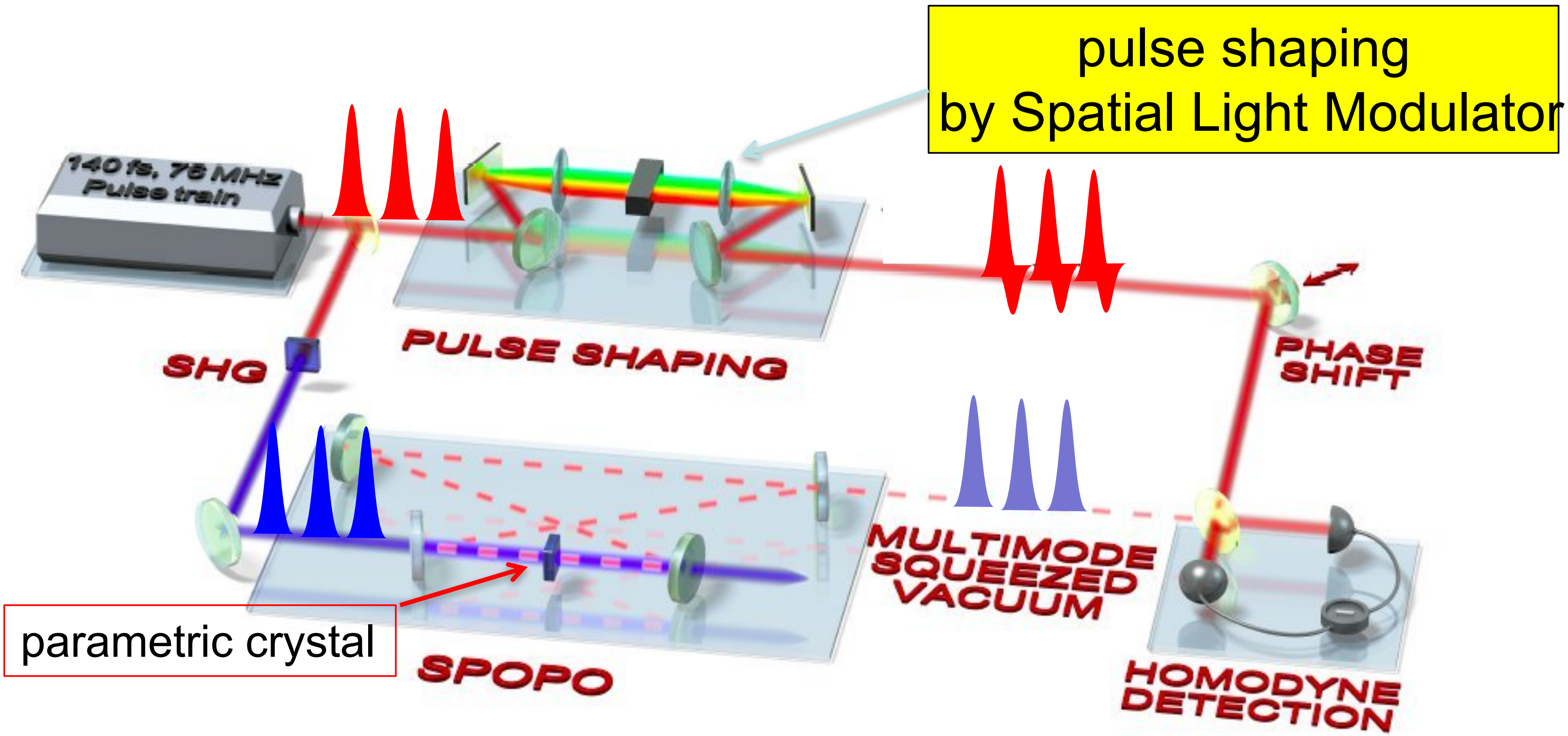


Eigenmodes: trains of pulses with Hermite-Gauss temporal shapes



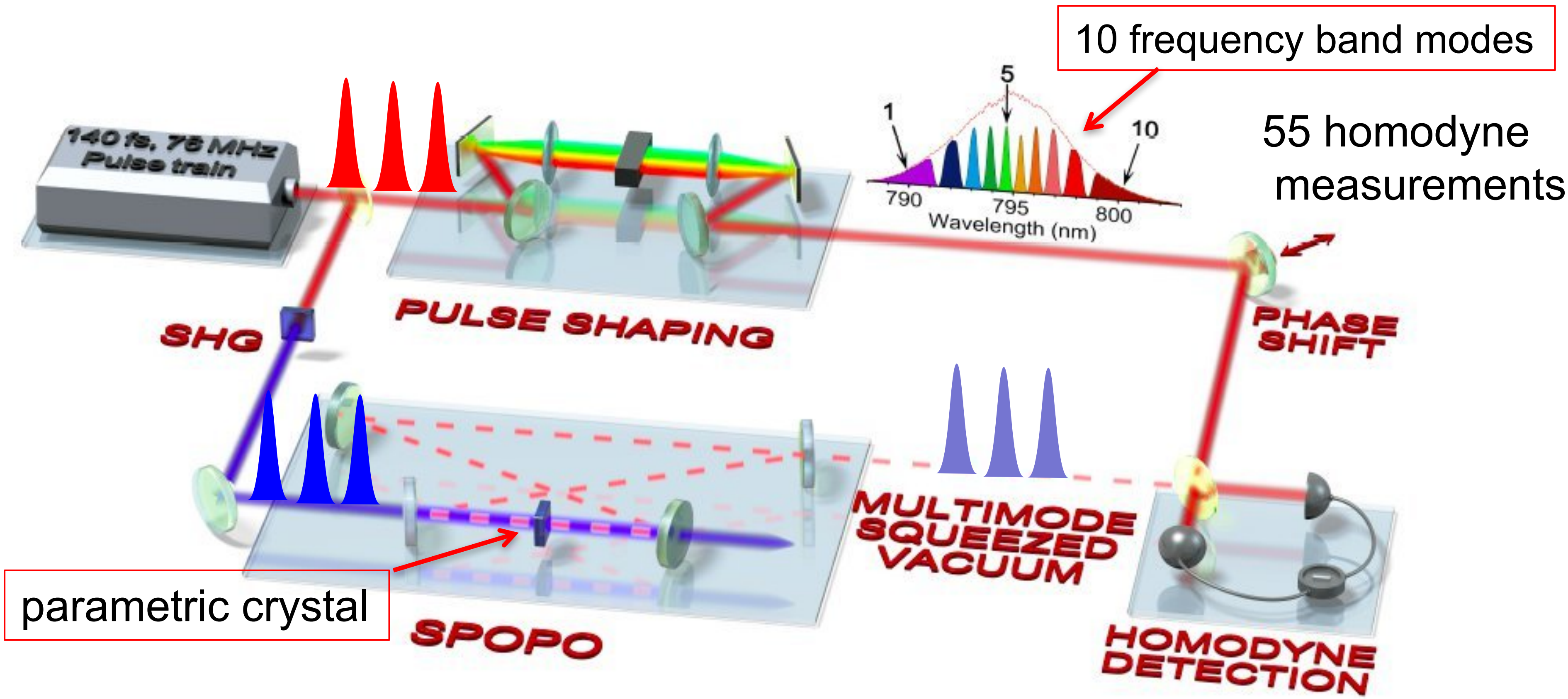
HG_0 HG_1 HG_2

Experimental set-up



O. Pinel et al, Phys. Rev. Letters **108,** 083601 (2012)
J. Roslund et al, Nature Photonics, **8**, 109 (2014)
R. Medeiros de Araujo et al, Phys Rev A**89,** 053828 (2014)
Yan Cai et al, Nature Com **8**, 15645 (2017)

Experimental set-up



O. Pinel et al, Phys. Rev. Letters **108,** 083601 (2012)
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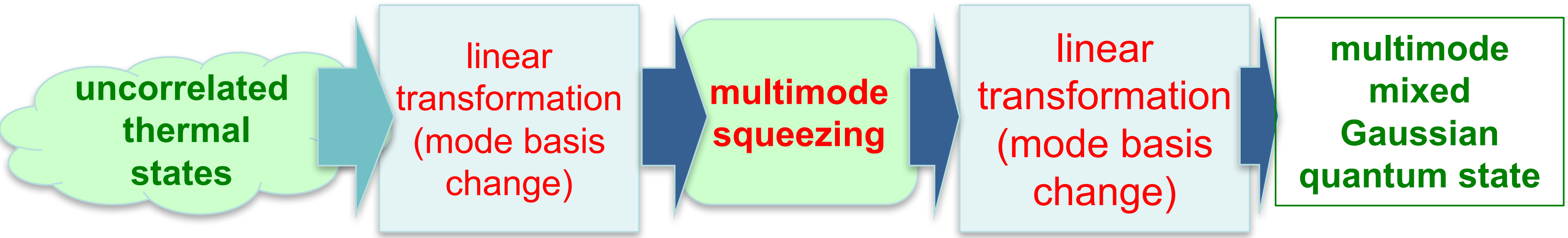
experimental value of the 20 by 20 covariance matrix

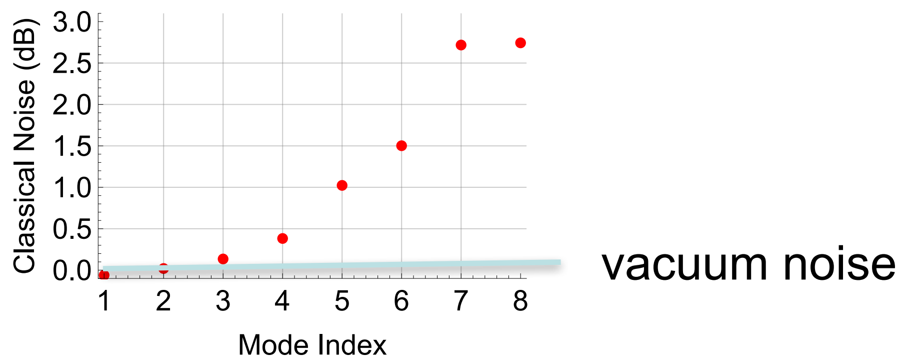
x quadratures *p* quadratures

$$
\begin{pmatrix}
\begin{pmatrix}
1.66 & 0.25 & 0.09 & 0.08 & -0.03 & -0.05 & -0.08 & -0.11 & -0.42 & -1.23 \\
0.25 & 1.33 & 0.19 & 0.15 & 0.06 & -0.06 & -0.17 & -0.33 & -0.59 & -0.51 \\
0.09 & 0.19 & 1.22 & 0.07 & -0.01 & -0.08 & -0.21 & -0.34 & -0.44 & -0.20 \\
0.08 & 0.15 & 0.07 & 1.08 & -0.02 & -0.12 & -0.21 & -0.24 & -0.25 & -0.07 \\
-0.03 & 0.06 & -0.01 & -0.02 & 0.92 & -0.13 & -0.14 & -0.11 & -0.09 & 0.01 \\
-0.05 & -0.06 & -0.08 & -0.12 & -0.13 & 0.89 & -0.07 & 0.02 & 0.02 & 0.04 \\
-0.08 & -0.17 & -0.21 & -0.21 & -0.14 & -0.07 & 1.02 & 0.12 & 0.10 & 0.20 \\
-0.11 & -0.33 & -0.34 & -0.24 & -0.11 & 0.02 & 0.12 & 1.14 & 0.29 & 0.22 \\
-0.42 & -0.59 & -0.44 & -0.25 & -0.09 & 0.02 & 0.10 & 0.29 & 1.36 & 0.40 \\
-1.23 & -0.51 & -0.20 & -0.07 & 0.01 & 0.04 & 0.20 & 0.22 & 0.40 & 1.88
\end{pmatrix} & [0] \\
[0] &
\begin{pmatrix}
1.66 & 0.25 & 0.09 & -0.02 & 0.04 & 0.00 & 0.01 & 0.09 & 0.39 & 1.19 \\
0.25 & 1.33 & 0.19 & 0.05 & 0.13 & 0.12 & 0.14 & 0.28 & 0.50 & 0.53 \\
0.09 & 0.19 & 1.22 & 0.17 & 0.21 & 0.28 & 0.32 & 0.37 & 0.39 & 0.26 \\
-0.02 & 0.05 & 0.17 & 1.31 & 0.35 & 0.38 & 0.37 & 0.33 & 0.26 & 0.12 \\
0.04 & 0.13 & 0.21 & 0.35 & 1.38 & 0.47 & 0.43 & 0.31 & 0.22 & 0.10 \\
0.00 & 0.12 & 0.28 & 0.38 & 0.47 & 1.42 & 0.42 & 0.30 & 0.19 & 0.10 \\
0.01 & 0.14 & 0.32 & 0.37 & 0.43 & 0.42 & 1.34 & 0.27 & 0.21 & 0.08 \\
0.09 & 0.28 & 0.37 & 0.33 & 0.31 & 0.30 & 0.27 & 1.30 & 0.22 & 0.15 \\
0.39 & 0.50 & 0.39 & 0.26 & 0.22 & 0.19 & 0.21 & 0.22 & 1.36 & 0.40 \\
1.19 & 0.53 & 0.26 & 0.12 & 0.10 & 0.10 & 0.08 & 0.15 & 0.40 & 1.88
\end{pmatrix}
\end{pmatrix}
$$

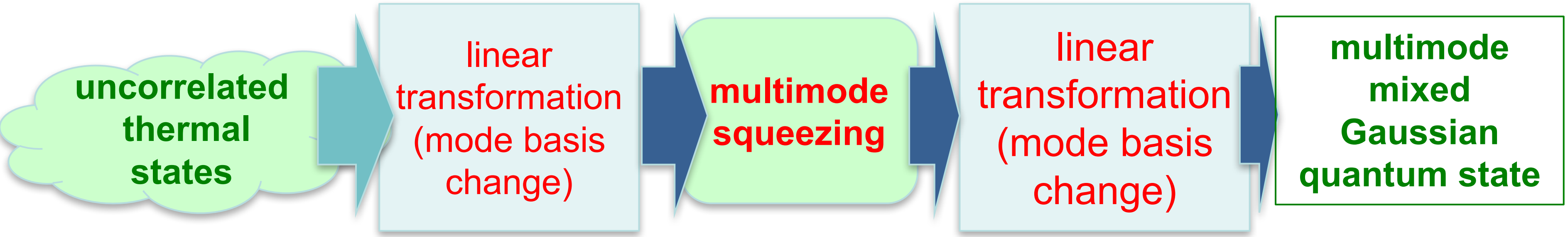
characterization of the multimode state
by Williamson Bloch Messiah reduction



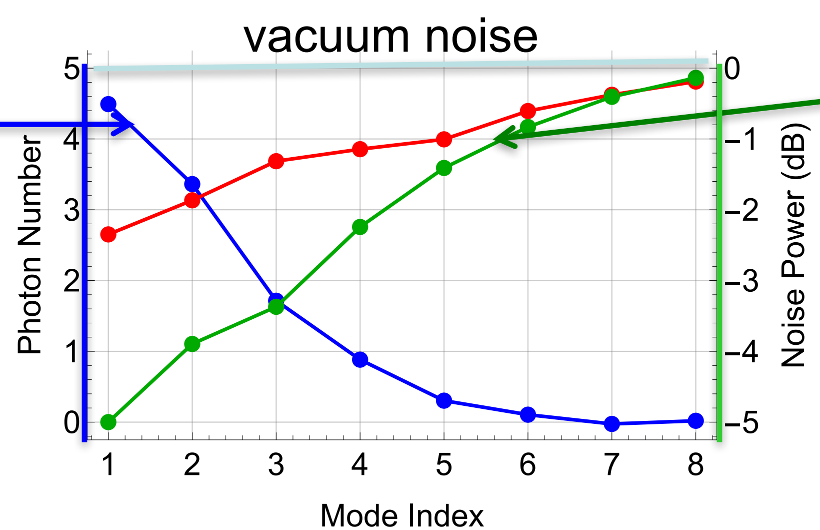
excess noise of input thermal modes:



characterization of the multimode state by Williamson Bloch Messiah reduction

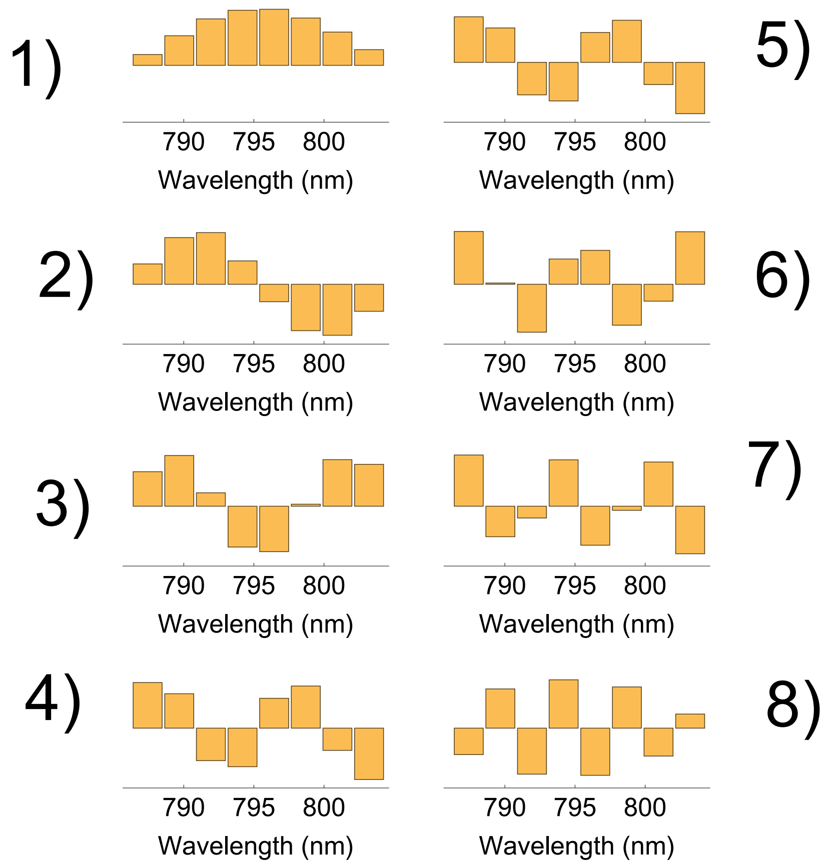


mean photon number in each eigenmode of the **coherence matrix**

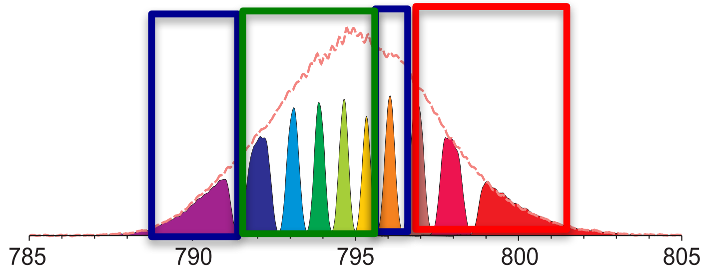


squeezing in the different modes

frequency shape of the squeezed eigenmodes



multi-partite entanglement



115 974
possible multipartitions
with 10 bands

multipartite optimal entanglement witnesses from covariance matrix:

J. Sperling and W. Vogel, Phys. Rev. Lett. **111**, 110503 (2013).

S. Gerke, J. Sperling, W. Vogel, Y. Cai, J. Roslund, N. Treps, C. Fabre, Phys. Rev. Letters **114,** 050501 (2015)

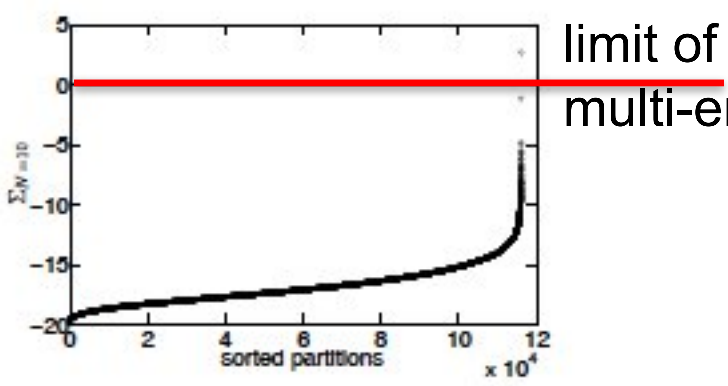
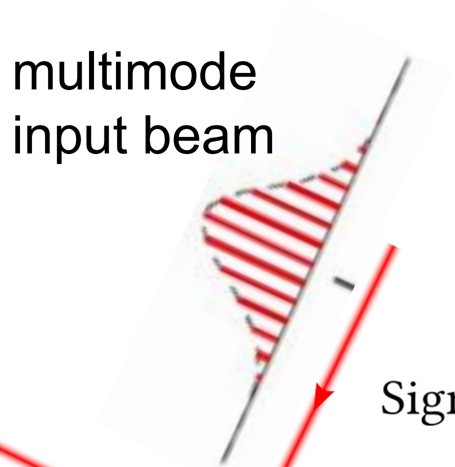


FIG. 3. The verified entanglement for all 115974 nontrivial partitions – sorted by significance Σ – for the 10-mode frequency-comb Gaussian state.

all 115 974 multipartitions are entangled !

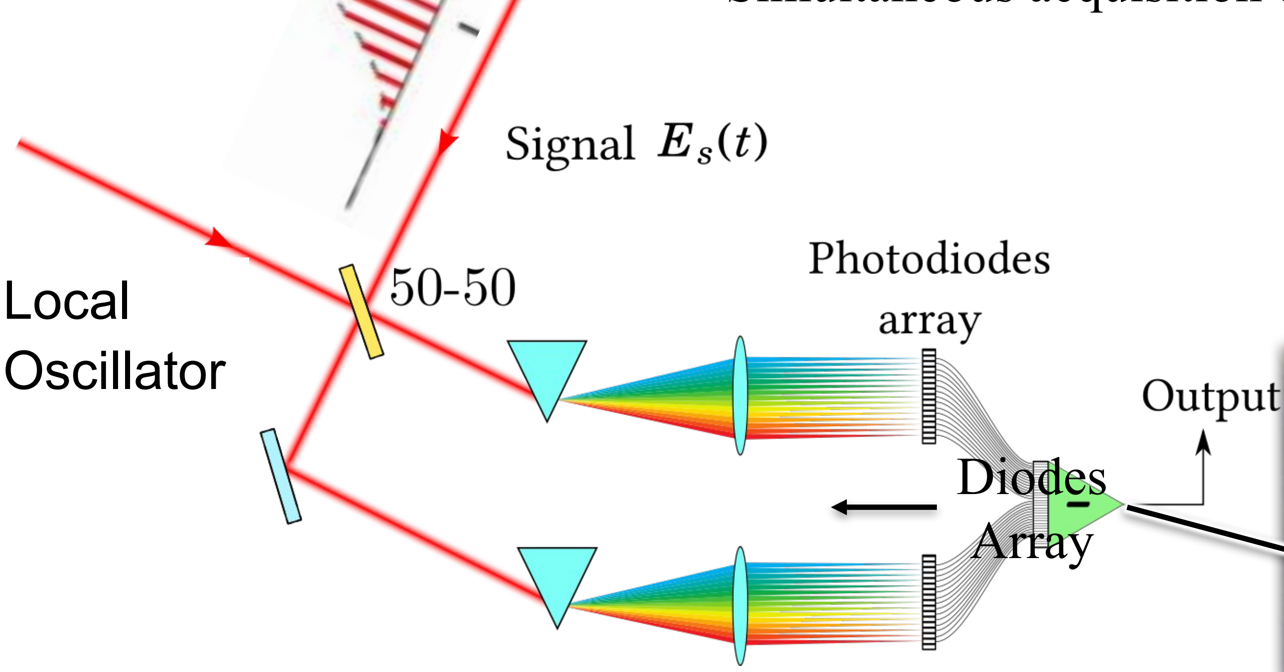
Multiplexed detection and processing

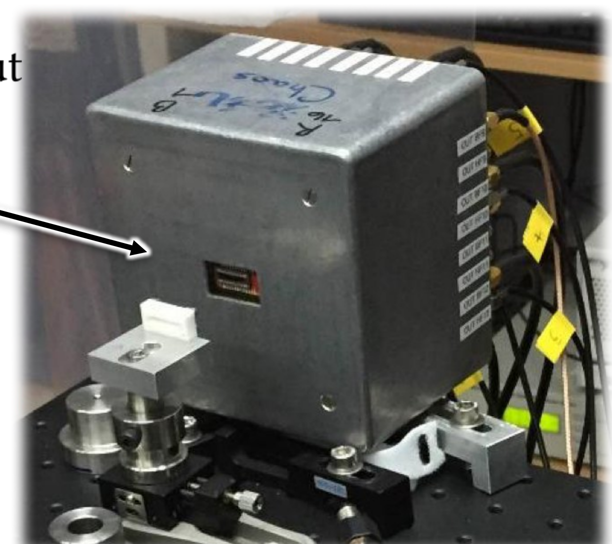
A technique to measure simultaneously the different modes



the Multi-Pixel Homodyne Detection

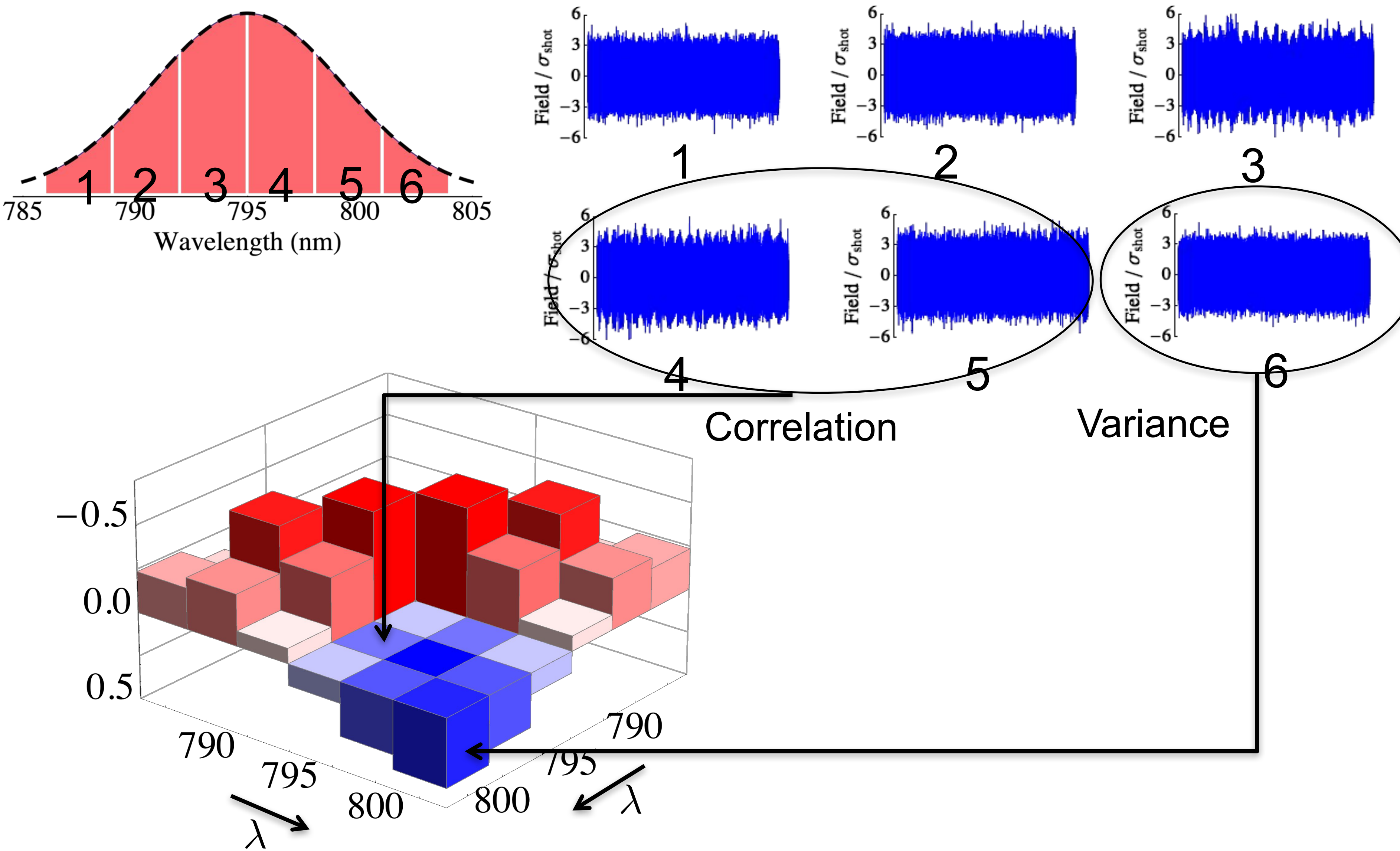
- Simultaneous acquisition of frequency bands



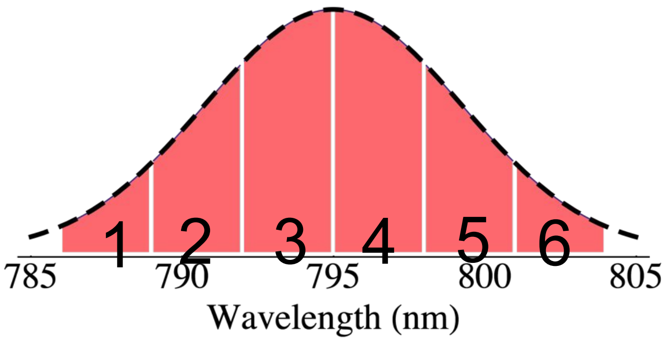


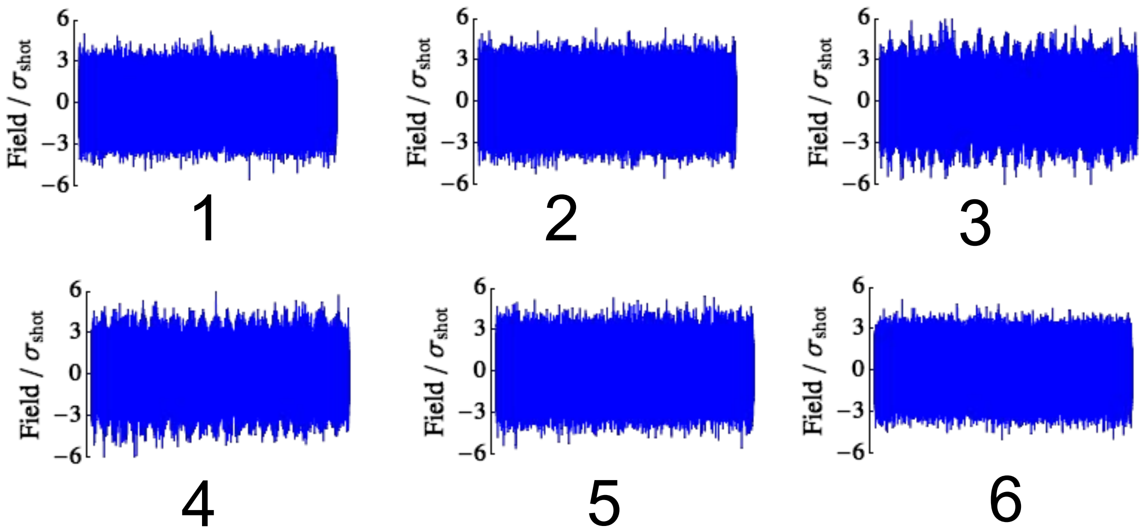
M.Beck, PRL 84 5748 (2000);
S. Armstrong et al, Nat. Comm. 3, 1026 (2012).
G. Ferrini et al New J Phys 15, 093015 (2013)

Multiplexed determination of (part of) quadrature covariance matrix



Multiplexed determination of quadrature of any mode



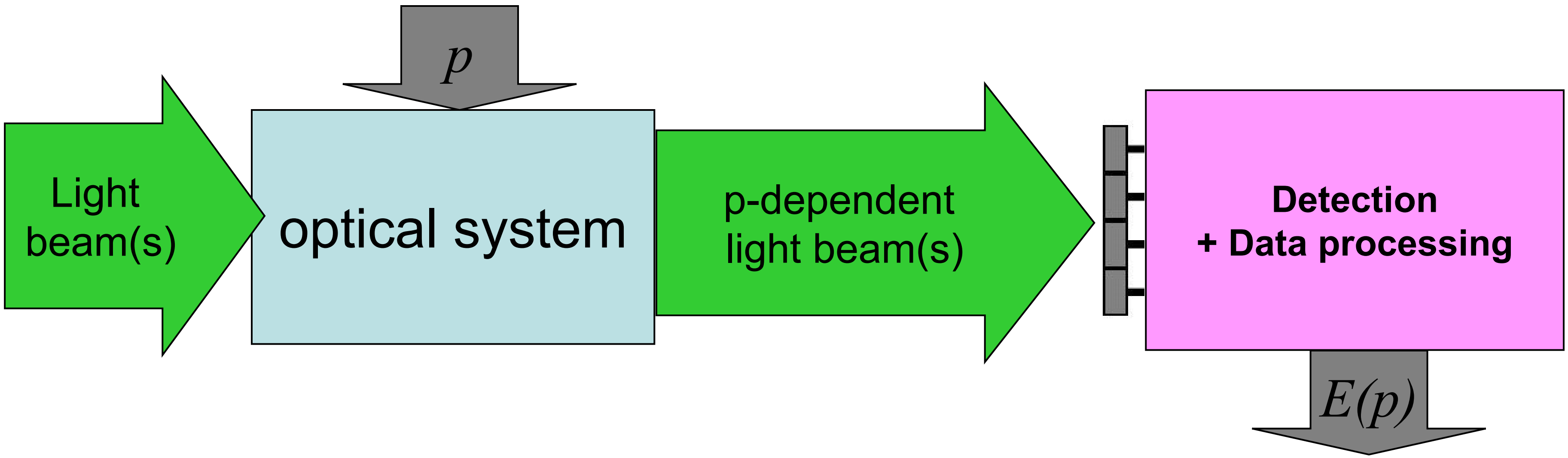


as a mode is a linear combination of « frexel » modes, computer calculates the same linear combination of the recorded fluctuations

permits « live » access
to the quadrature fluctuations of any mode,

Mode optimization in parameter estimation

General scheme for estimating a parameter p using information carried by light



Estimator of p

What is the smallest measurable variation of p around value p_0, for a given mean photon number ?

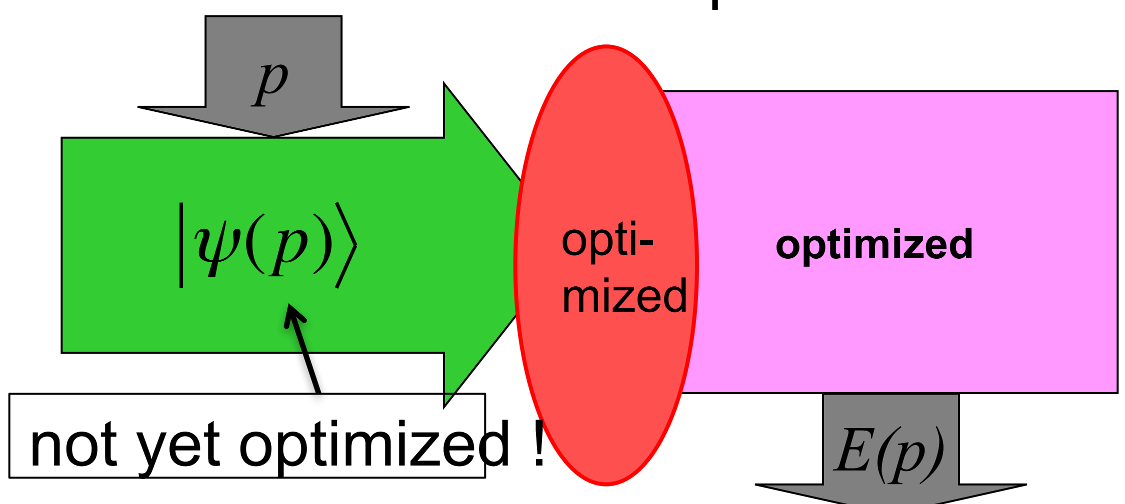
Quantum Cramer Rao Bound

The minimum variance of any (unbiased) estimator is given by the inverse of quantum Fischer information

the bound is optimized over

- all the possible techniques of photodetection
- all the possible data processing strategies

It depends only on the characteristics of the quantum state of light



choice of light state $|\psi(p)\rangle$?

best choice **for an experimentalist** in Quantum Optics:

- large mean photon number N

  quantum limits scale as $1/N^x$

- non-classical state of light

- **multimode state**

$$\hat{\mathbf{E}}^{+}(\mathbf{r},t) = \sum_n (\hat{X}_n + i\hat{Y}_n)\,\mathbf{f}_n(\mathbf{r},t)$$

choice of multimode quantum light state

choice of mode shape

a practical choice : the multimode Gaussian pure state

includes **a wide class of non-classical states**

- single and multimode squeezed states
- Einstein Podolsky Rosen (EPR) state
- multipartite quadrature entangled state
- coherent state with high photon number

$$\boxed{\left\langle N \right\rangle \approx 10^{15}}$$

optimization over :

- squeezing and entanglement
- number of modes
- the spatio-temporal shape of modes

excludes states which are « more quantum »,
but not scalable to very large N value

Useful mode 1: the « illumination mode »

$$u_{mean}(x,y,t,p)=\frac{1}{\sqrt{N}}\left\langle \psi\left(p,t\right)\middle|\hat{E}^{(+)}(x,y)\middle|\psi(p,t)\right\rangle$$

contains the spatio-temporal dependence of the mean field.

for a small variation around p_0 :

$$u_{mean}(x,y,z,t,p)\simeq u_{mean}(x,y,z,t,p=p_0)+(p-p_0)\frac{\partial u_{mean}}{\partial p}$$

information about p related to

$$\frac{\partial u_{mean}}{\partial p}$$

Useful mode 1: the « illumination mode »

$$u_{mean}(x,y,t,p) = \frac{1}{\sqrt{N}} \left\langle \psi\left(p,t\right) \middle| \hat{E}^{(+)}(x,y) \middle| \psi(p,t) \right\rangle$$

contains the spatio-temporal dependence of the mean field.

for a small variation around p_0 :

$$u_{mean}(x,y,z,t,p) \simeq u_{mean}(x,y,z,t,p=p_0) + (p-p_0)\frac{\partial u_{mean}}{\partial p}$$

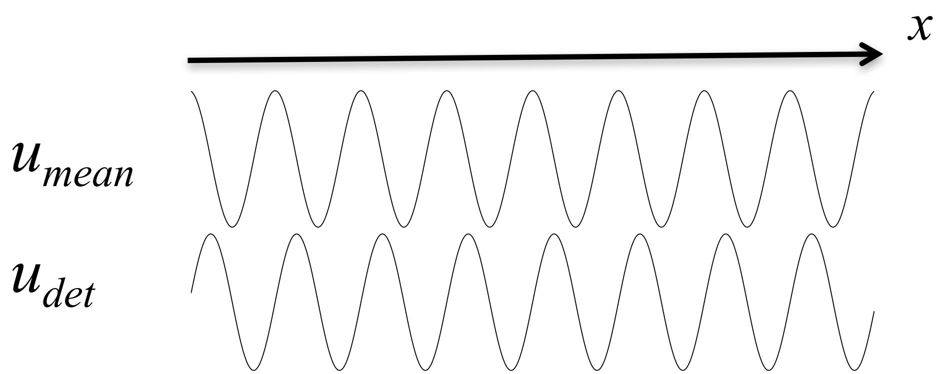
Useful mode 2: the «detection mode »

$$u_{\det}(x,y,t) = p_c \left.\frac{\partial u_{mean}}{\partial p}\right|_{p=p_0}$$

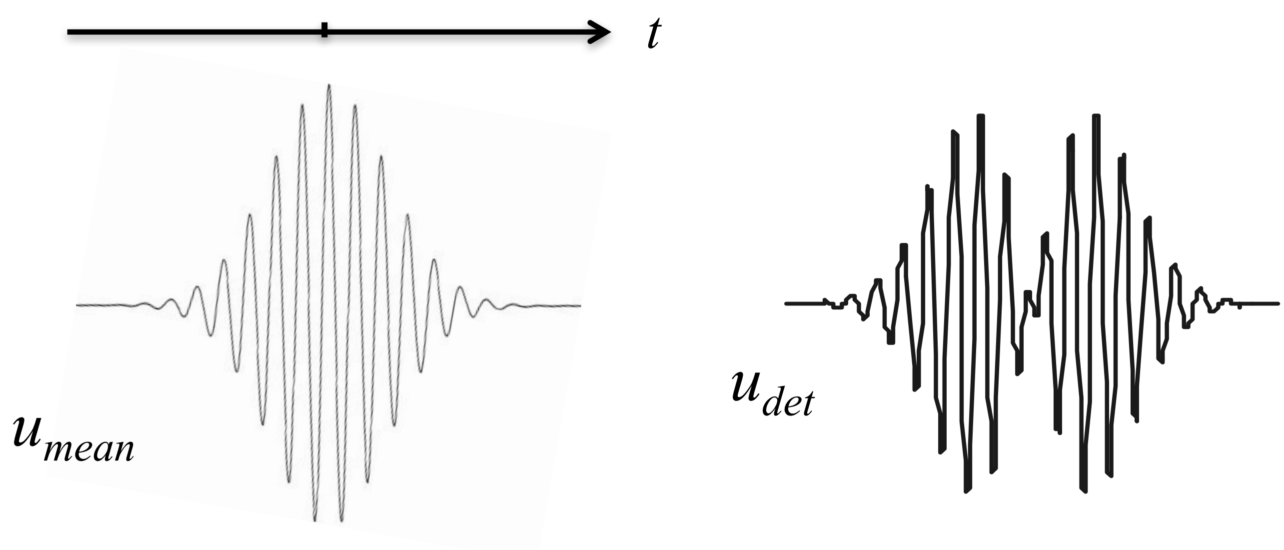
normalizing factor

examples of detection mode

Phase shift



Time delay
of light pulse



Quantum Cramer Rao bound for Gaussian pure states

p-sensitivity

expression in the high N limit:

$$\Delta p_{QCRb} = \frac{p_c}{2\sqrt{N}}\sigma_n$$

« shot noise »

noise term

Quantum Cramer Rao bound for Gaussian pure states

p-sensitivity

expression in the high N limit:

$$\Delta p_{QCRb} = \frac{p_c}{2\sqrt{N}}\sigma_n$$

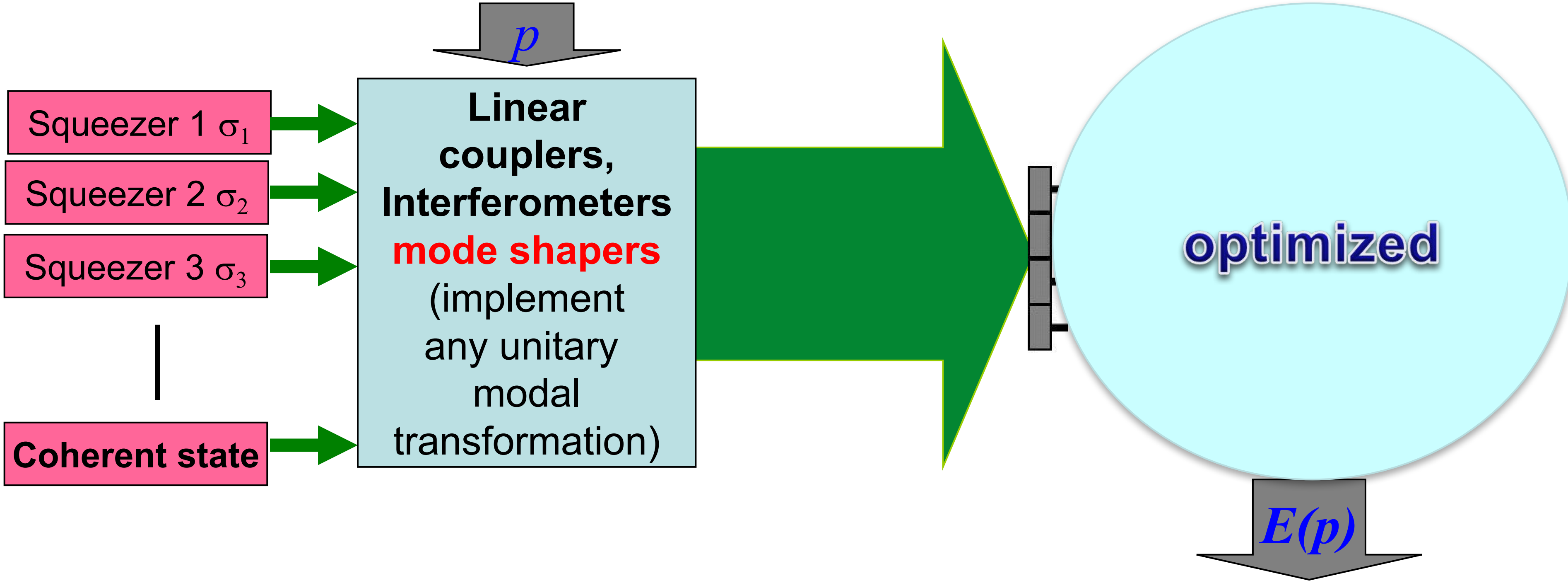
« shot noise »

noise term

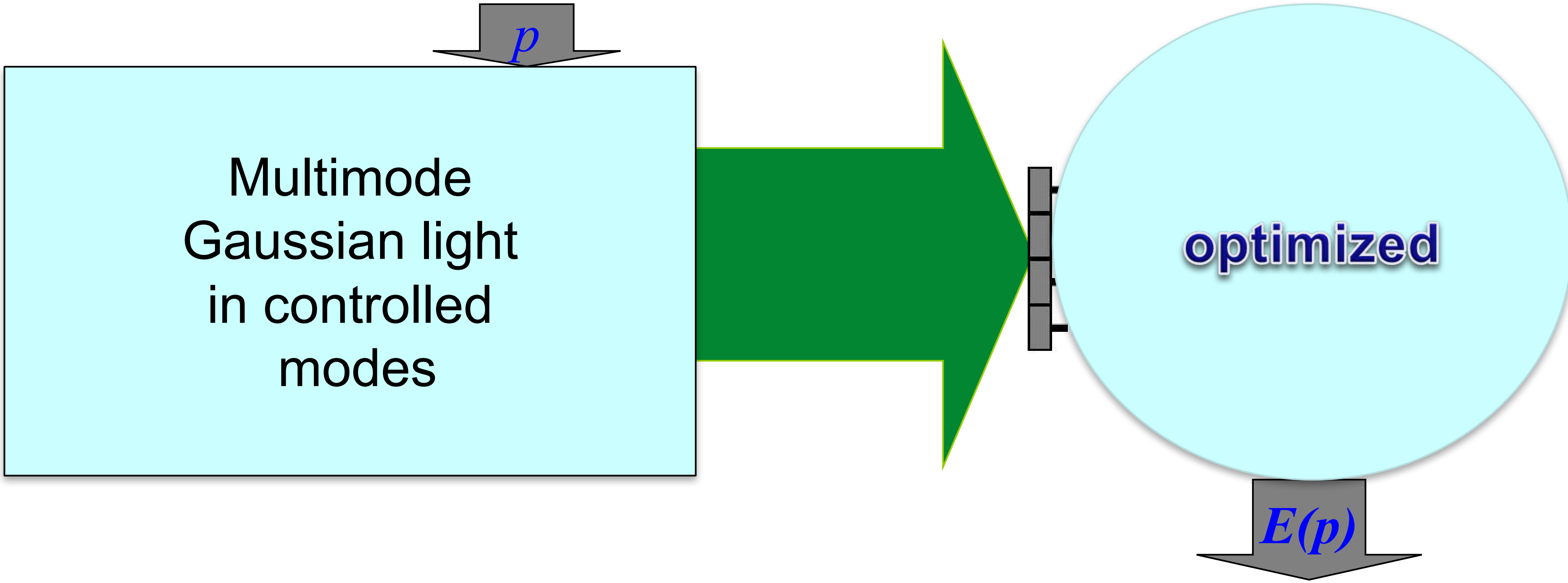
Diagonal element of the inverse covariance matrix in the **detection mode** u_{det}

σ_n depends only on the noise of the detection mode

Quantum Cramér Rao bound using experimentalist's non-classical Gaussian resources



Quantum Cramér Rao bound using experimentalist's non-classical Gaussian resources



$$\Delta p_{CRb} = \frac{p_c}{2\sqrt{N}}\sqrt{\sigma_{\min}}$$

obtained when the detection mode contains best squeezed mode

$$\sigma_{\min} = Min\{\sigma_1, \sigma_2, \ldots, \sigma_s\}$$

Instructions for the experimentalist

To get the lowest possible Quantum Cramer Rao bound

- Minimize scaling factor p_c

- Put maximum power in coherent mode

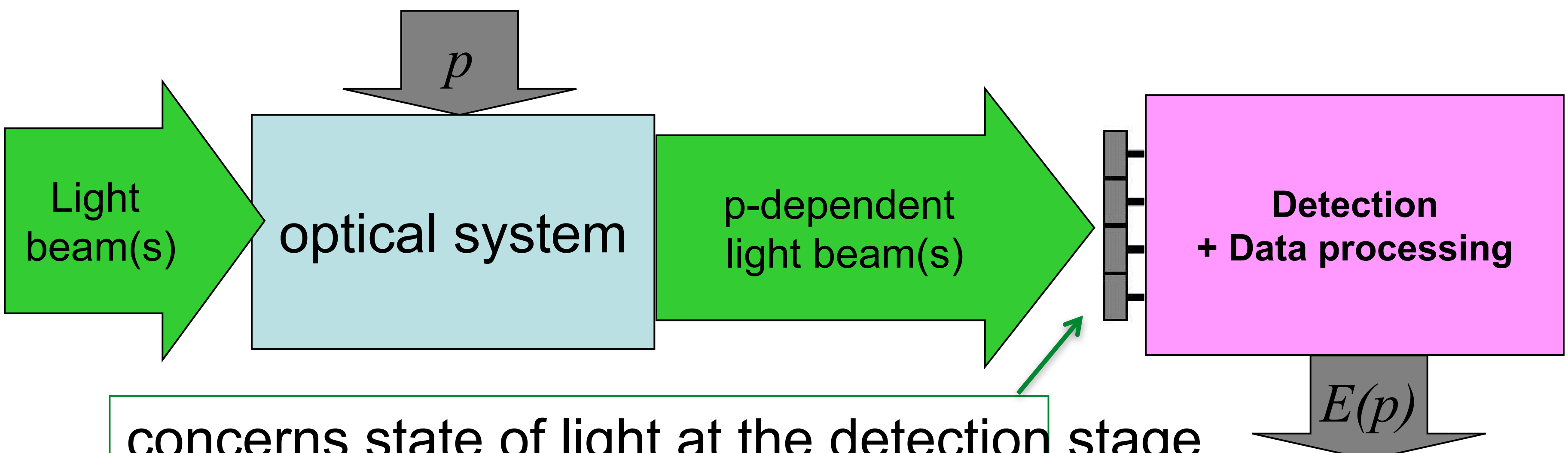
- Squeeze one mode only

squeezing is not « additive »

- Squeeze the right mode

squeeze the detection mode

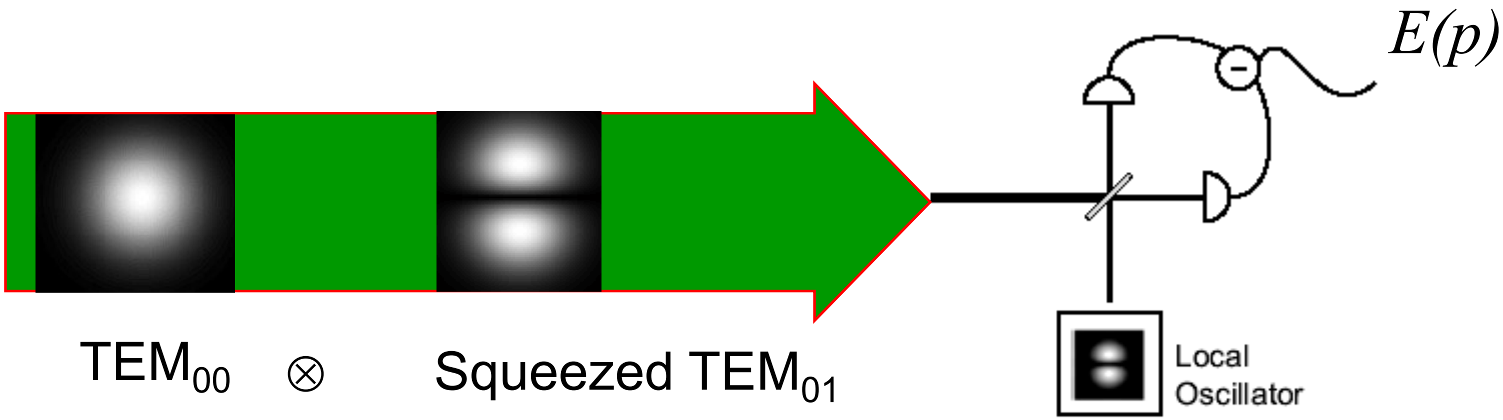
- Do not entangle detection mode with other modes



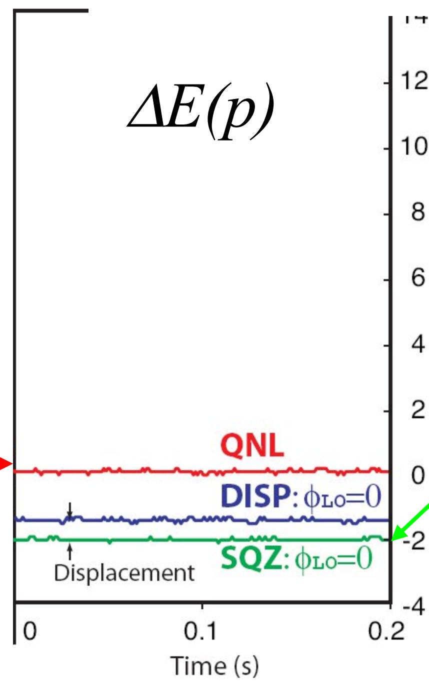
concerns state of light at the detection stage

Experiments:
parameter estimation below the standard quantum limit

1) Estimation of transverse beam displacement



M. Lassen et al.
Phys. Rev Letters
98, 083602 (2007)

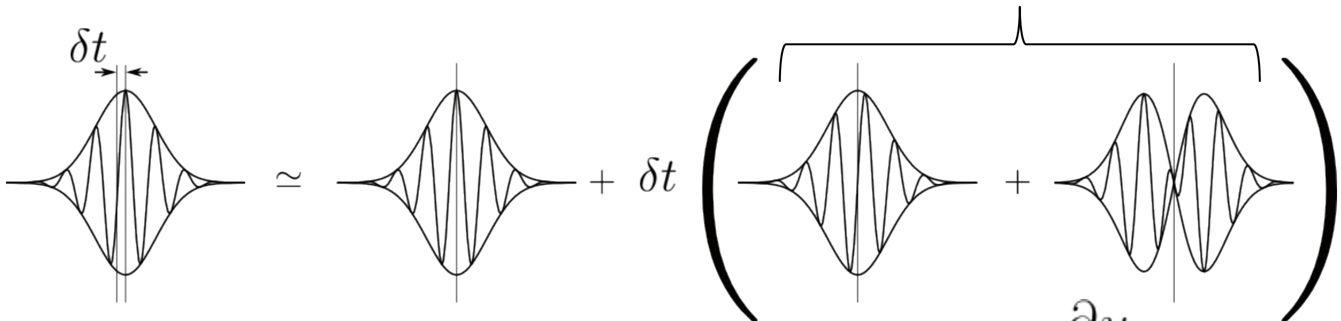


Standard Cramer Rao bound

Reduced noise floor using squeezed TEM_{10}

2) Estimation of a time delay

Detection mode for δt



$$u(t+\delta t) \simeq u(t) + \delta t \, \underbrace{\left(\ldots \right)}_{w_L(t) \propto \frac{\partial u}{\partial t}(t)}$$

Mean field mode

Detection mode

2 terms in the detection mode:

- **Carrier**: sensitive to phase delay, accessed by phase measurement
- **Envelope**: sensitive to group delay, accessed by time of flight measurement

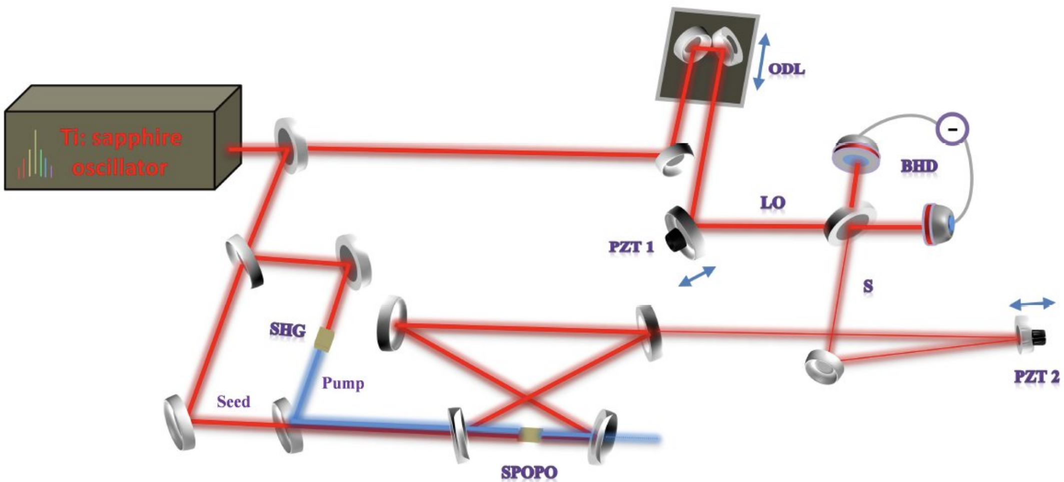
using both information: $$(\delta t)_{\min} = \frac{1}{2\sqrt{N}\sqrt{\omega_0^2 + \Delta\omega^2}}$$

Recent experiment

Quantum improved measurement of time transfer

Shaofeng Wang[1,2], Xiao Xiang[1,2], Nicolas Treps[3], Claude Fabre[3], Tao Liu[1,2], Shougang Zhang[1,2], Ruifang Dong[1,2,*]
[1] *Key Laboratory of Time and Frequency Primary Standards,*
National Time Service Center, Chinese Academy of Sciences, Xi'an, 710600, China
[2] *University of Chinese Academy of Sciences, Beijing, 100049, China and*
[3] *Laboratoire Kastler Brossel, Sorbonne Université,*

(unpublished)



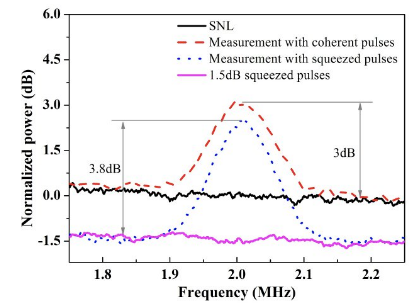


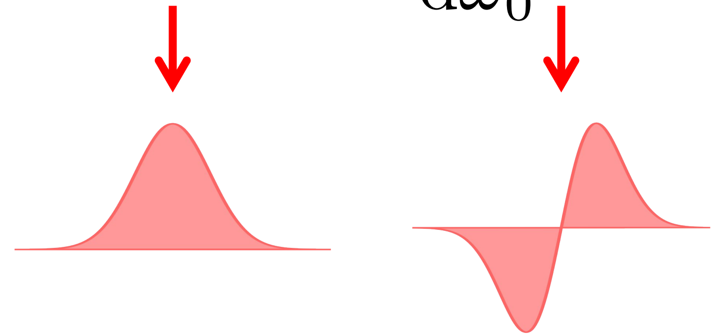
FIG. 3. Measured BHD signals with coherent and squeezed pulses as the signal arm respectively. The applied modulation voltage on PZT2 was set to 1.7 V. The data acquisition was implemented by using a spectrum analyzer with a RBW of 100 kHz and a VBW of 30 Hz.

estimation made in the coherent phase measurement case

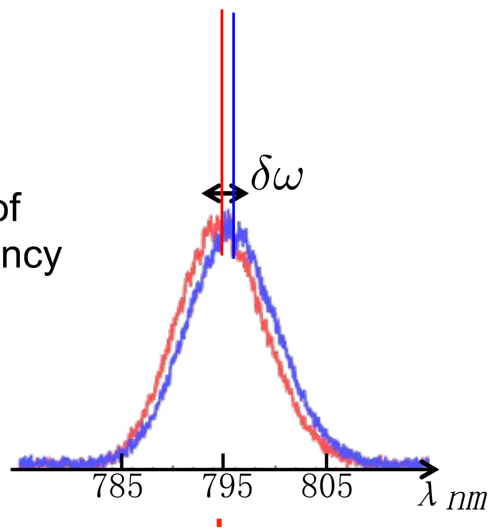
improvement of estimation from 2.8 10^{-20} s to 2.4 10^{-20} s

3) Estimation of a spectral shift

$$E(\omega, \delta\omega) = E(\omega) + \delta\omega \frac{\mathrm{d}E}{\mathrm{d}\omega_0}(\omega)$$



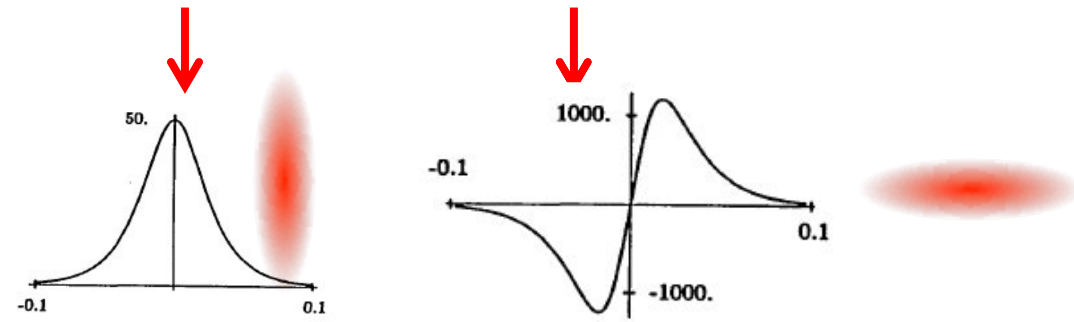
Displacement of Central Frequency

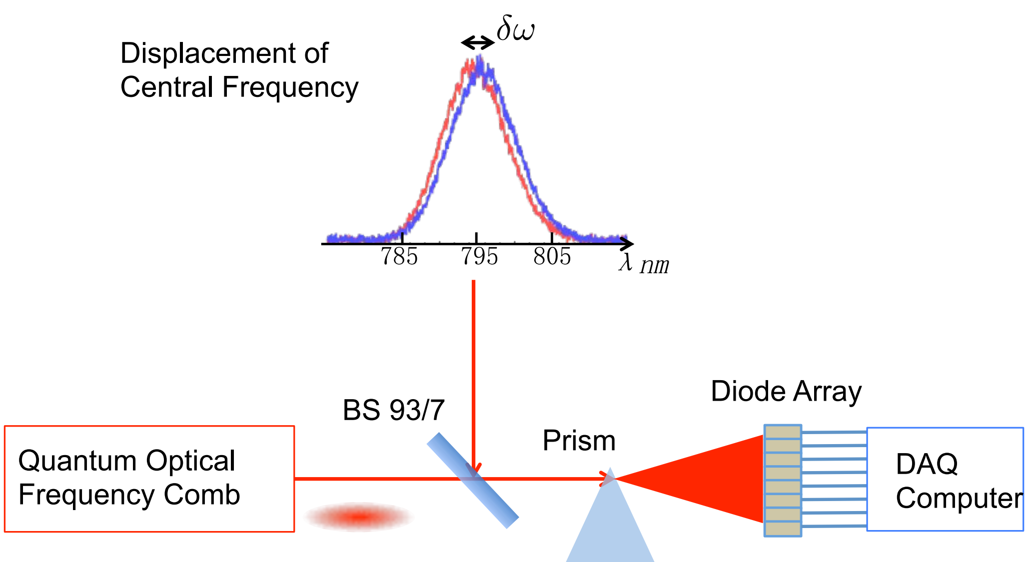


Experiment

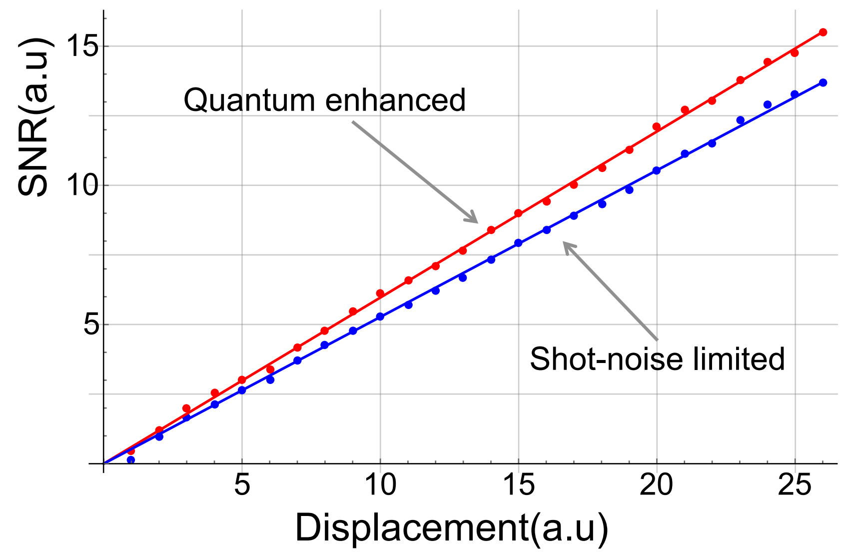
J. Roslund, Y. Cai,
C. Fabre and N. Treps
A Quantum spectrometer.
in preparation

multimode squeezed state produced
by parametric down conversion of mode locked laser:





A



B

~20% increase in sensitivity $61\,\mathrm{kHz}/\sqrt{\mathrm{Hz}}$ ⇨ $50\,\mathrm{kHz}/\sqrt{\mathrm{Hz}}$

Conclusion

- in many applications, the shape of the mode(s)
  in which a quantum state lives
  is as important as the quantum state itself

- possibility of change of mode basis
  opens many possibilites

- multiplexed detection permits to extract simultaneous
  information about different modes
  without physically extracting the modes

- manipulating multimode quantum states of light
  is a good starting point for up-scalable
  quantum information processing,
  in particular for measurement based quantum computing

Quantum state in mode basis change

the same quantum state $|\Psi\rangle$ has different expressions in different mode bases

two-mode example:

$$|\Psi\rangle = |1 : \mathbf{f}_1 \, , \, 1 : \mathbf{f}_2\rangle$$

factorized

$$|\Psi\rangle = |2 : \mathbf{g}_+ \, , \, 0 : \mathbf{g}_-\rangle - |0 : \mathbf{g}_+ \, , \, 2 : \mathbf{g}_-\rangle$$

entangled

with $\mathbf{g}_\pm = (\mathbf{f}_1 \pm \mathbf{f}_2)/\sqrt{2}$