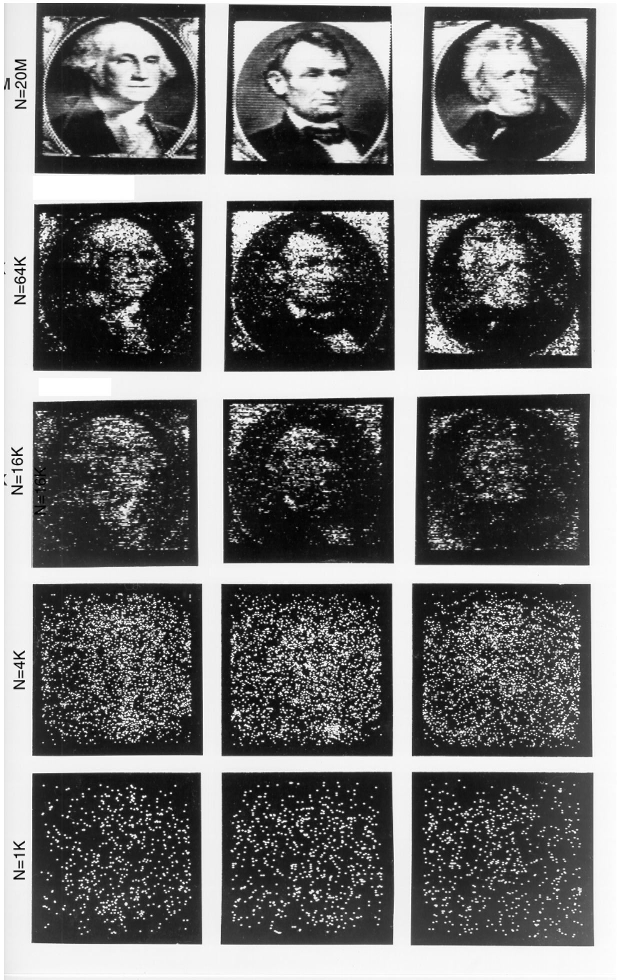
Quantum temporal imaging

Mikhail Kolobov

Laboratoire PHLAM, Université de Lille, FRANCE

Joint work with Giuseppe Patera, Dmitri Horoshko, and Junheng Shi

Quantum imaging



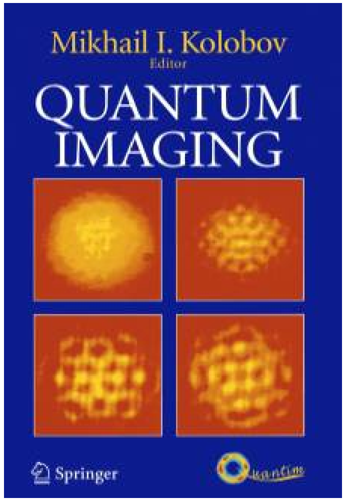
Topics:

- ✓ *noiseless image amplification*
- ✓ *ghost imaging*
- ✓ *quantum limits of resolution*
- ✓ *entangeled images*
- ✓ *image teleportation*

European projects:

QUANTIM (2001-2004)

HIDEAS (2008-2013)

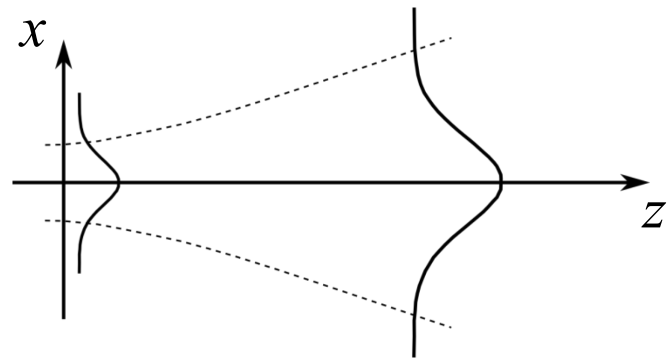


Space-time duality

Parallels between beam diffraction and temporal dispersive pulse broadening

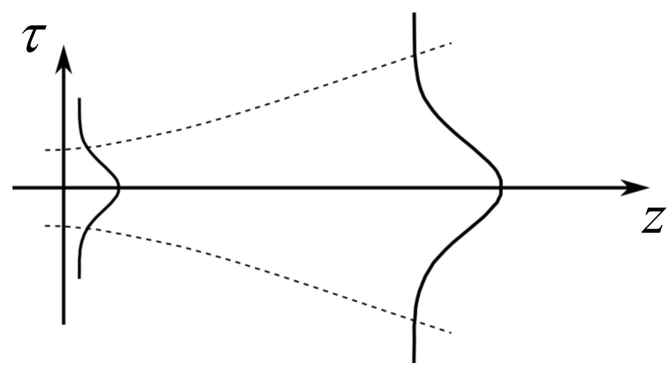
Space: diffraction in 1D

$$\frac{\partial A(z,x)}{\partial z} = -\frac{i}{2k}\frac{\partial^2 A(z,x)}{\partial x^2}$$



Time: temporal dispersive broadening

$$\frac{\partial A(z,\tau)}{\partial z} = i\frac{\beta_2}{2}\frac{\partial^2 A(z,\tau)}{\partial \tau^2}$$



with

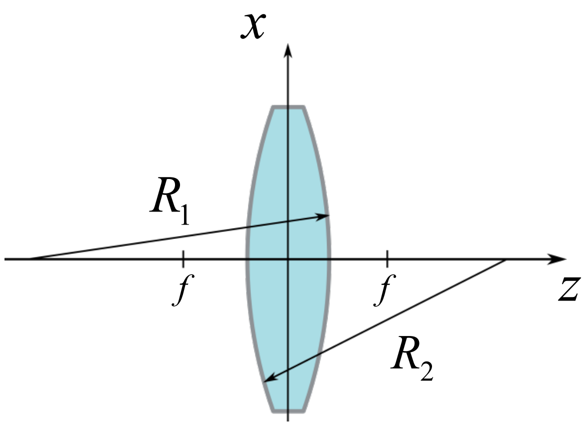
$\tau = t - t/v_g$ - retarded time

$v_g = \left(\frac{\partial k}{\partial \omega}\right)^{-1}$ - group velocity

$\beta_2 = \frac{\partial^2 k}{\partial \omega^2}$ - Group Velocity Dispersion (GVD)

Space lenses & time lenses

Space:

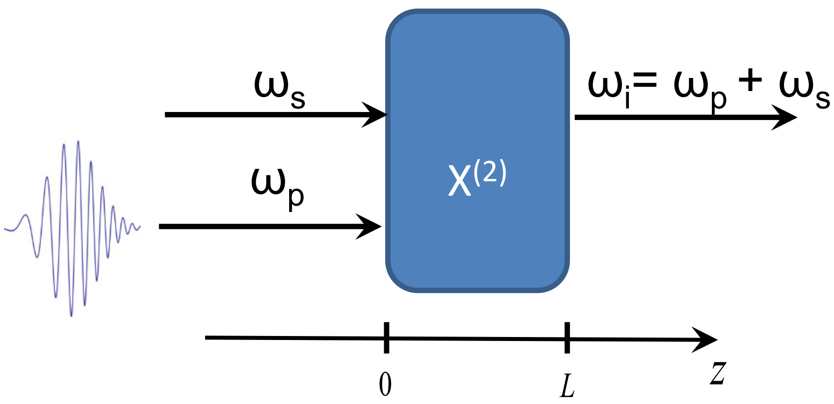


$$T_{lens}(x) = \mathrm{e}^{i\frac{k}{2\mathrm{f}}x^2}$$

f - focal distance

$$f = \frac{1}{(n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

Time:



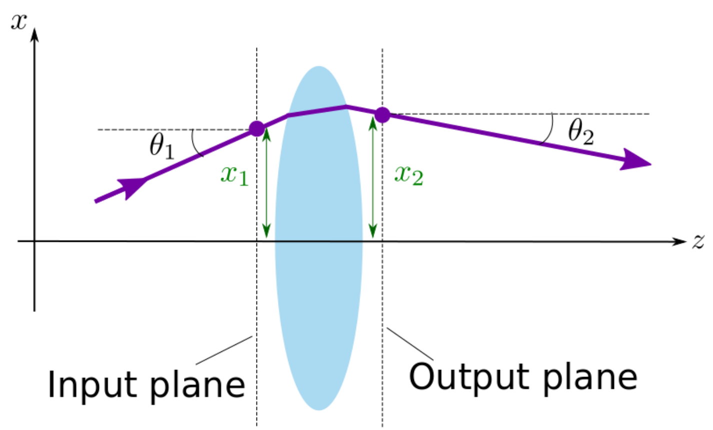
$$T_{lens}(\tau) = \eta \mathrm{e}^{i\frac{\tau^2}{2D_\mathrm{f}}}$$

D_f – group delay dispersion (GDD)

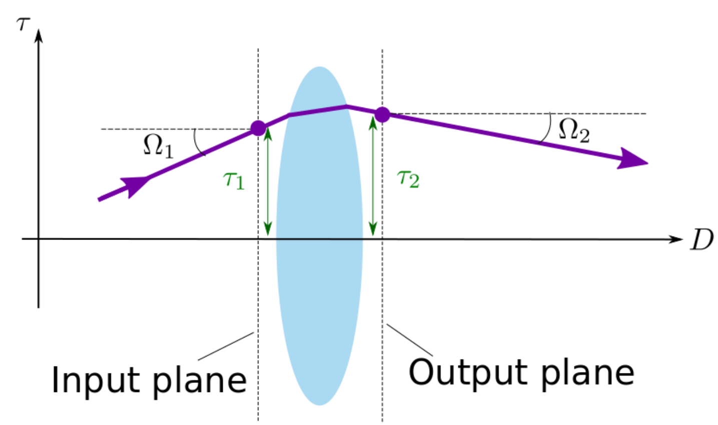
η - efficiency

Space rays & time rays

Space:



Time:



Space ray transform matrix (ABCD):

$$\begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix} = M \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix} \quad M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

- free-space propagation, distance d

$$M = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

- thin lens, focal distance f

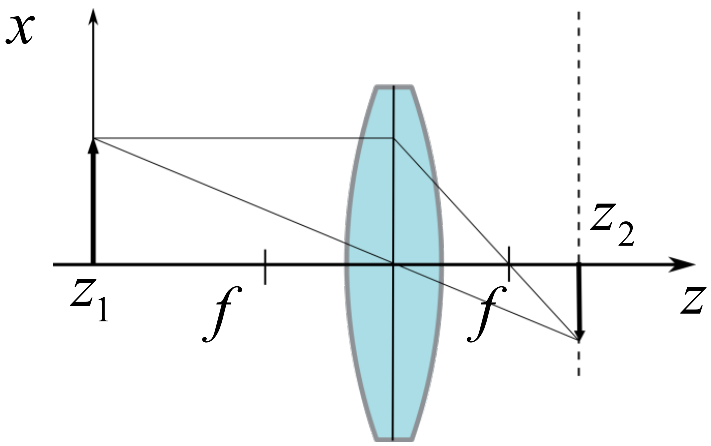
$$M = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

Works for time rays as well with:

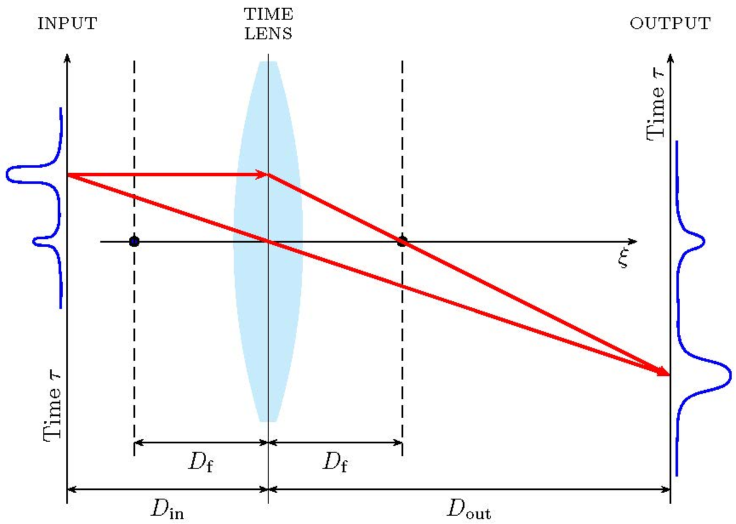
1) $z \rightarrow D = \beta_2 \xi$ [s^2]
2) more possibilities due to β_2 of both signs

Space imaging vs time imaging

Space:



Time:



Imaging conditions:

$$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}$$

$$M=-\frac{z_2}{z_1}$$

$$\frac{1}{D_{\text{in}}}+\frac{1}{D_{\text{out}}}=\frac{1}{D_{\text{f}}}$$

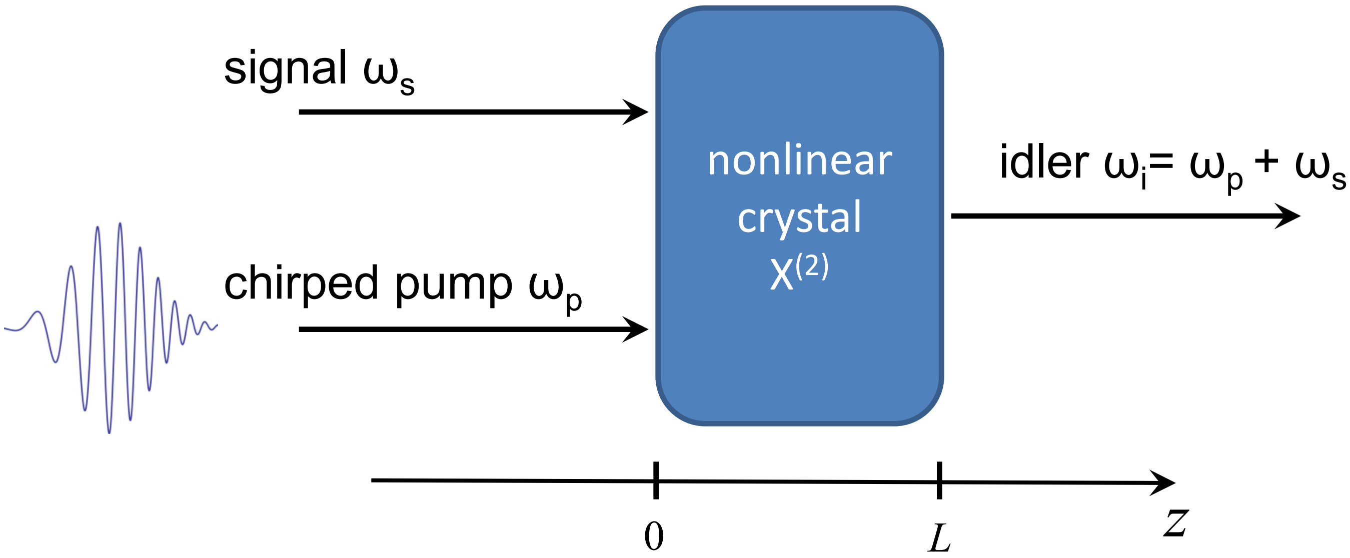
$$M=-\frac{D_{\text{out}}}{D_{\text{in}}}$$

with:

$$D_{\text{in}}=\beta_2^{(1)}z_1 \qquad D_{\text{out}}=\beta_2^{(2)}z_2$$

Parametric time lens

Sum-frequency generation process (SFG)



Pump: $a_p(t) = A_p(t)e^{i\phi_p(t)}$ undepleted

<u>Assumptions</u>:

- neglect Group Velocity Dispersion (GVD)
- group velocity v_g the same for signal and idler

Classical solution

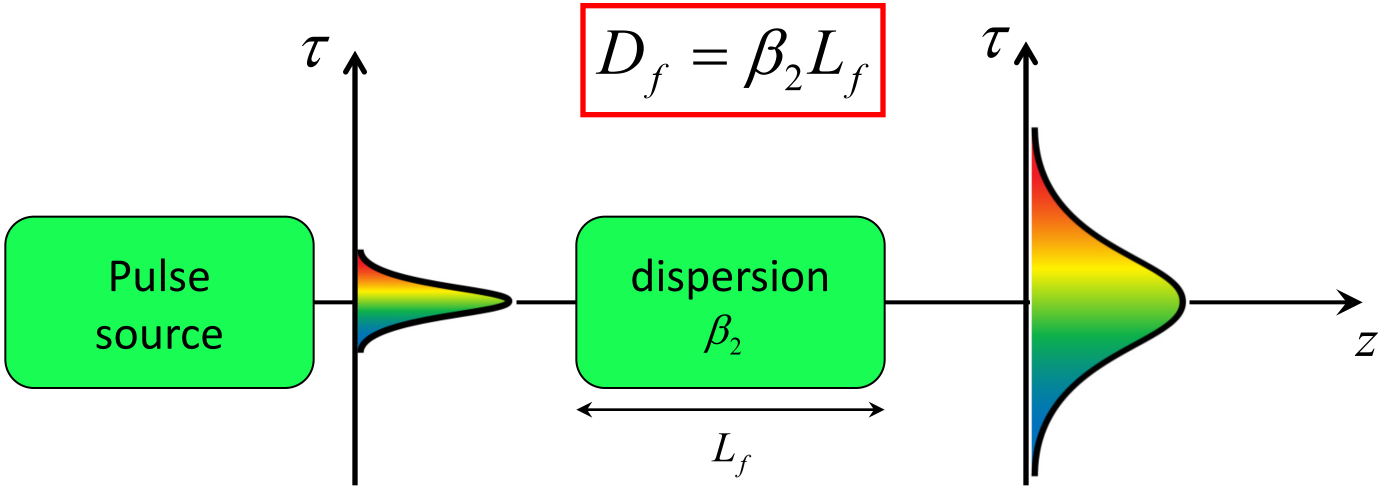
$$a_i(L,\tau) = -s(\tau)e^{i\phi_p(\tau)}a_s(0,\tau)$$ $$\tau = t - z/v_g$$

$$s(\tau) = \sin[gA_p(\tau)L]$$ Conversion efficiency $|s(\tau)| < 1$

Time lens: $$\phi_p(\tau) = \tau^2/2D_f$$ Quadratic time dependence or chirp

Chirp: propagate a short pulse in dispersive medium of length L_f with GVD $\beta_2 = \frac{\partial^2 k}{\partial \omega^2}$

$$D_f = \beta_2 L_f$$



Quantum solution

Classical solution with $|s(\tau)|<1$ is not applicable for quantum fields;
Does not satisfy commutation relations

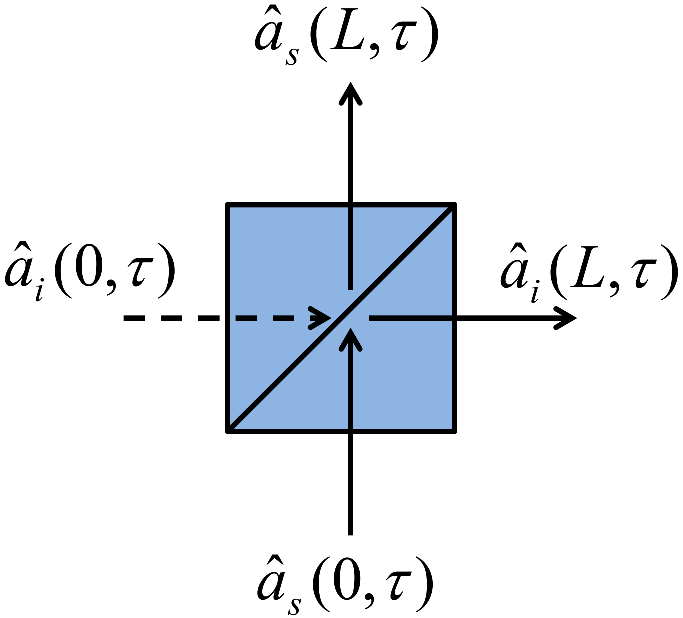
$$\hat{a}_s(L,\tau)=c(\tau)\hat{a}_i(0,\tau)+s(\tau)e^{-i\phi_p(\tau)}\hat{a}_i(0,\tau)$$

$$\hat{a}_i(L,\tau)=-s(\tau)e^{i\phi_p(\tau)}\hat{a}_s(0,\tau)+c(\tau)\hat{a}_i(0,\tau)$$

- unitary transforamtion

- preserves comm. relations

- equivalent to a beam-splitter



Quantum solution

with reflection and transmission coefficients

$$s(\tau) = \sin[gA_p(\tau)L], \qquad c(\tau) = \cos[gA_p(\tau)L], \qquad c^2 + s^2 = 1$$

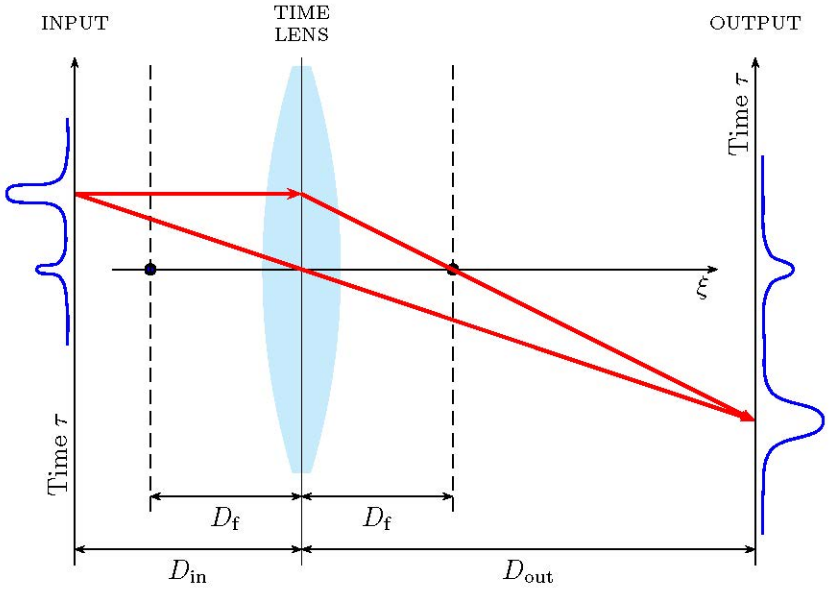
- if $|s(\tau)| < 1$ then the field $\hat{a}_i(0,\tau)$ brings the vacuum fluctuations into the output

- these vac. fluctuations are eliminated when

$$s(\tau) = 1 \qquad \Leftrightarrow \qquad \boxed{gA_p(\tau)L = \pi/2}$$

then $$\boxed{\hat{a}_i(L,\tau) = -e^{i\phi_p(\tau)}\hat{a}_s(0,\tau)}$$

Single-lens temporal imaging system



<u>Imaging condition</u>:

$$\frac{1}{D_{in}} + \frac{1}{D_{out}} = \frac{1}{D_f}$$

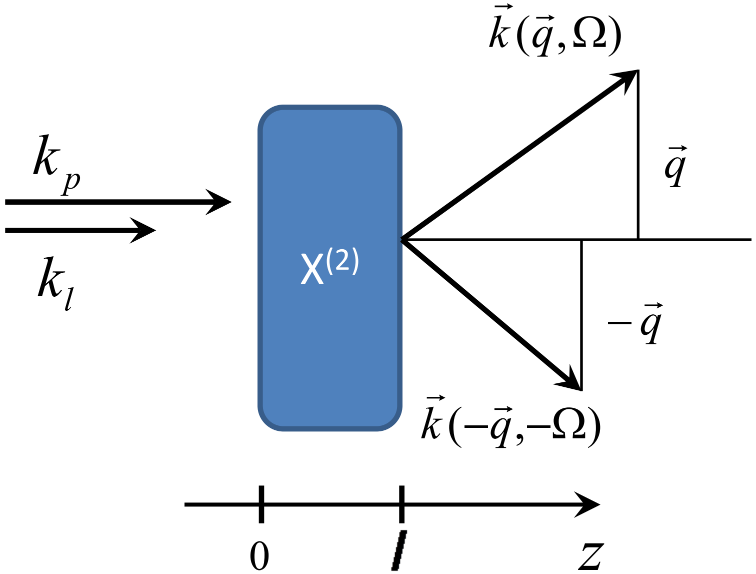
with magnification

$$M = -\frac{D_{out}}{D_{in}}$$

<u>Quantum imaging transform</u>:

$$\hat{a}_i'(\tau) = -\frac{1}{\sqrt{M}} e^{i\frac{\tau^2}{2MD_f}} \hat{a}_s(\tau/M)$$

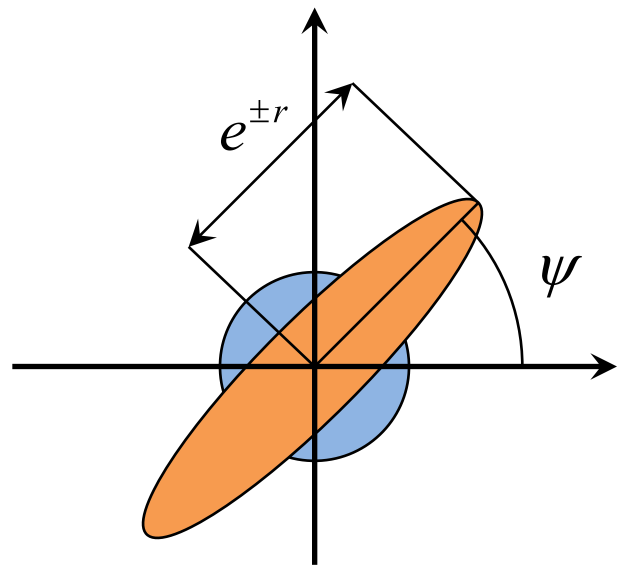
Temporally broadband squeezed input state



Optical Parametric Amplifier
(OPA)

Squeezing transform

$$\hat{a}_s(l,\Omega) = U(\Omega)\hat{a}_s(0,\Omega) + V(\Omega)\hat{a}_s^+(0,-\Omega)$$



$$\psi(\Omega) = \frac{1}{2}\arg\left[V(\Omega)/U(\Omega)\right]$$

$$\exp\left[\pm r(\Omega)\right] = \left|U(\Omega) \pm V(\Omega)\right|$$

Squeezing transform vs squeezed state

Squeezing transform is described by 4 real parameters:

$$r(\Omega) = \ln\left|U(\Omega) + V(\Omega)\right|$$ - squeezing parameter

$$\psi(0,\Omega) = \frac{1}{2}\arg\left[V(\Omega)/U(\Omega)\right]$$ - input phase

$$\psi(L,\Omega) = \frac{1}{2}\arg\left[U(\Omega)V(-\Omega)\right]$$ - output phase

$$\kappa(\Omega) = \frac{1}{2}\arg\left[U(\Omega)/U(-\Omega)\right]$$ - phase delay

Using these parameters we define the eigen field queadratures:

Squeezing transform vs squeezed state

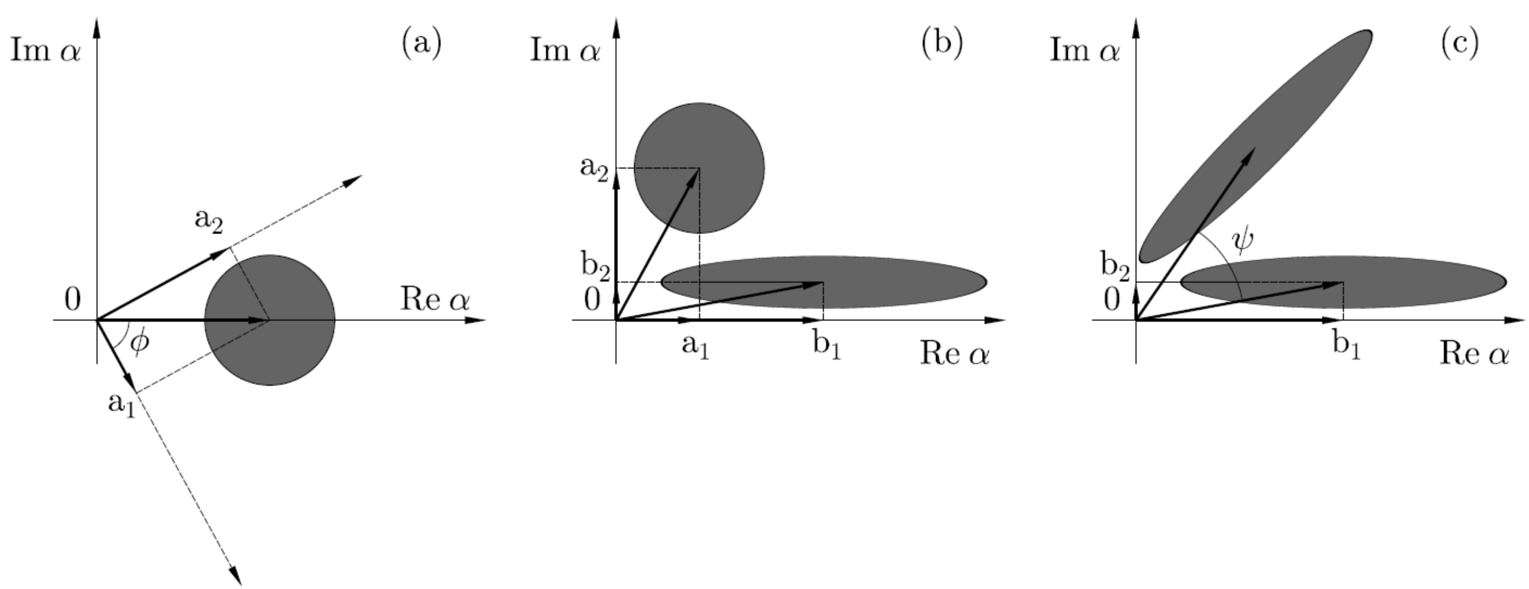
$$X_1(z,\Omega) = \hat{a}(z,\Omega)\exp[-i\psi(z,\Omega)] + \hat{a}^+(z,-\Omega)\exp[i\psi(z,\Omega)]$$

$$X_2(z,\Omega) = -i(\hat{a}(z,\Omega)\exp[-i\psi(z,\Omega)] - \hat{a}^+(z,-\Omega)\exp[i\psi(z,\Omega)])$$

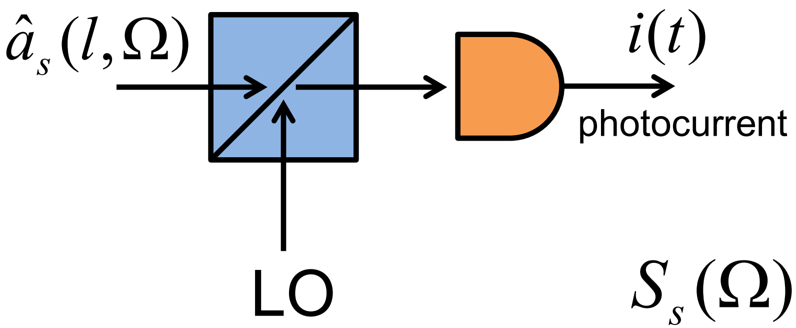
Squeezing transform:

$$X_j(z,\Omega) = \exp[\pm r(\Omega) + \kappa(\Omega)](X_j(z,\Omega)$$

upper(lower) sign corresponds to j=1(j=2)



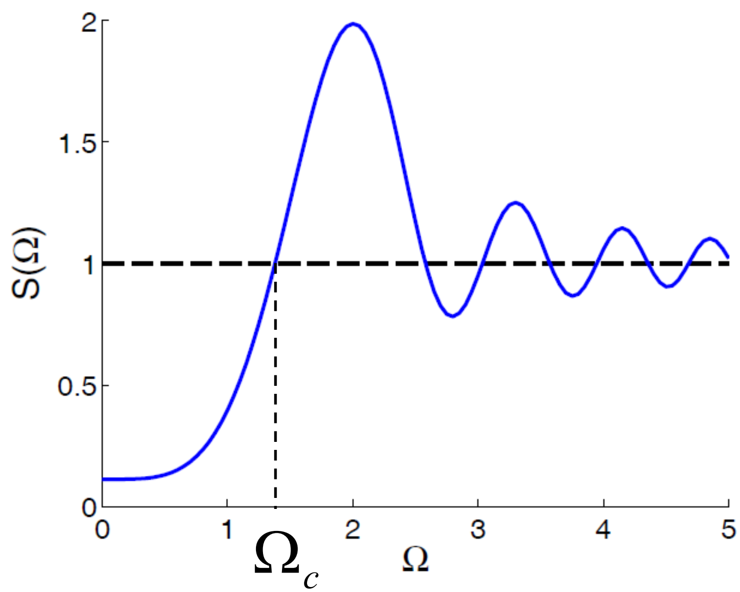
Squeezing spectrum before temporal lens



$$S_s(\Omega) = \cos^2[\theta(\Omega)]e^{2r(\Omega)} + \sin^2[\theta(\Omega)]e^{-2r(\Omega)}$$

$$\theta(\Omega) = \psi(\Omega) - \varphi$$

φ - the phase of the LO



$$\Omega_c = \left(\beta_2^{(c)} l\right)^{-1/2}$$

$\beta_2^{(c)}$ - GVD coeff. of the OPA

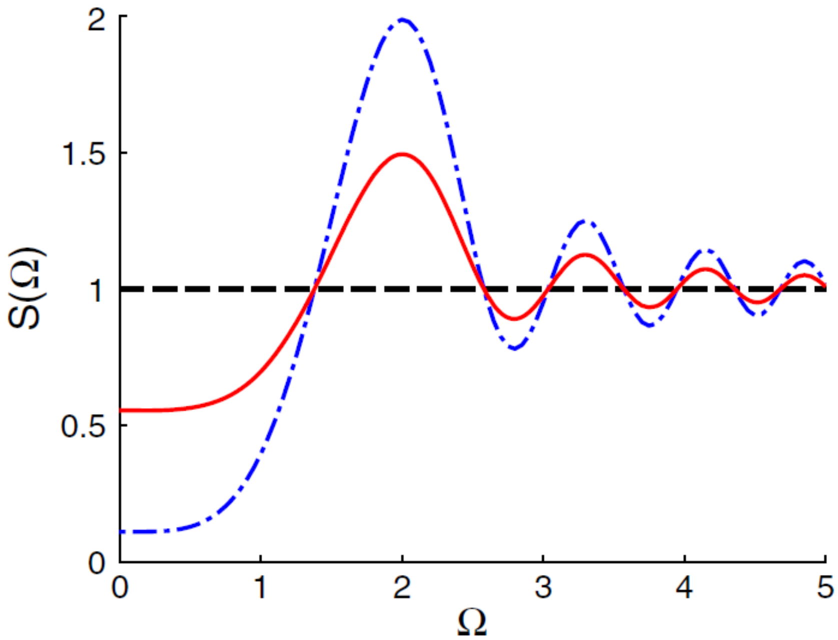
$\tau_c = \dfrac{\pi}{\Omega_c}$ - coherence time

For $T >> \tau_c$ - sub-Poissonian photocurrent

Squeezing spectrum after temporal lens

$$S_i(\Omega) = 1 - \eta + \eta\left[\cos^2\left[\theta(\Omega)\right]e^{2r(\Omega)} + \sin^2\left[\theta(\Omega)\right]e^{-2r(\Omega)}\right]$$

with $\eta = s^2(\tau)$



$\eta = 1/2$

- for $\eta < 1$ squeezing deteriorates

- for $\eta = 1$ squeezing spectrum after the lens is identical to that before

Squeezing spectrum after single lens: temporal imaging system

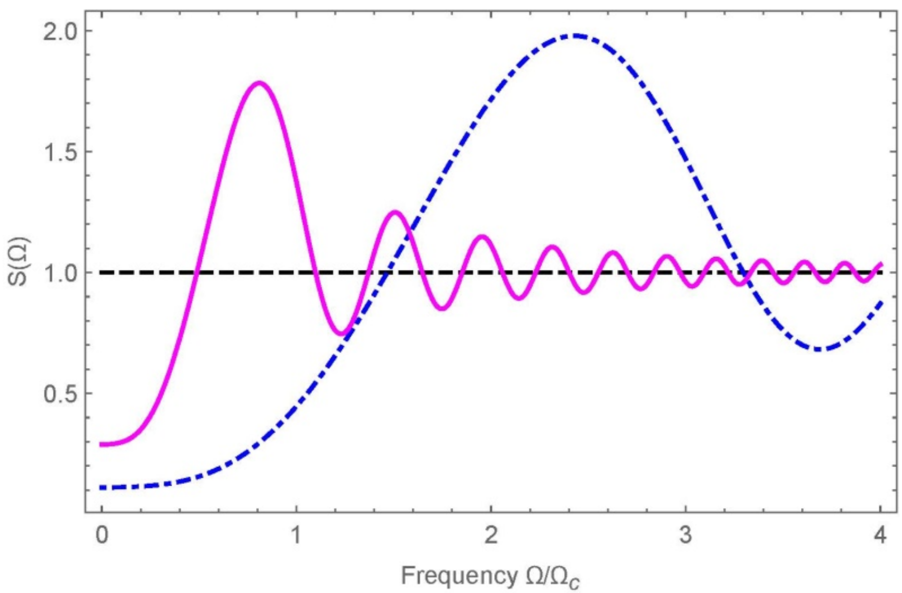
1) 4-f imaging scheme with $D_s = 2D_f$, $D_i = 2D_f$

$$S_s(\Omega)\Big|_{z=l} = S_i(\Omega)\Big|_{z=l+4L_f}$$

with $L_f = D_f / \beta_2$ equivalent focal length

Noiseless temporal imaging of squeezed light

2) Geometric imaging with magnification factor M. Example for M=-3, η=0.8

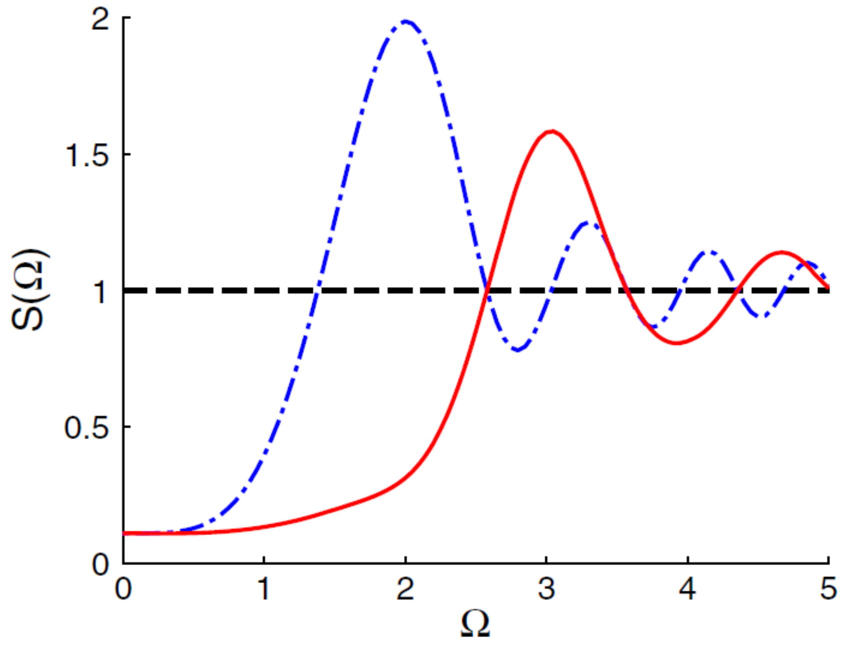


Partial compensation of frequency dispersion of the OPA

4-f imaging scheme of an object plane inside the OPA, i.e.

$$z = l + L_s^{(1)} \qquad \text{with} \qquad L_s^{(1)} = -l_{amp} \frac{\beta_2^{(c)}}{2\beta_2^{(1)}}$$

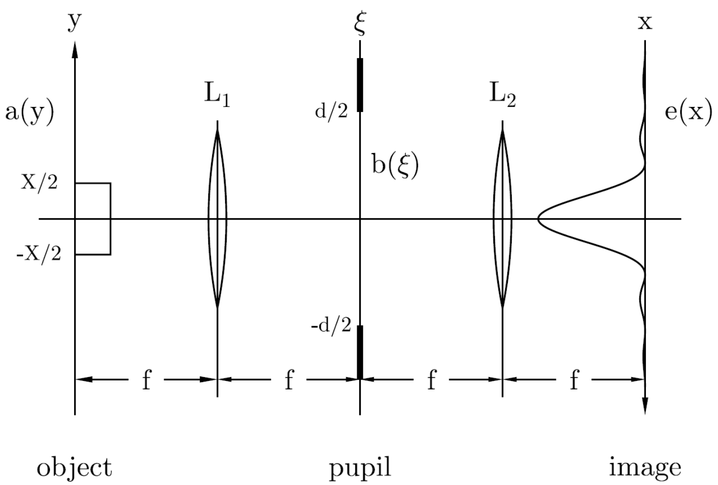
$$\boxed{S_s(\Omega)\Big|_{z=l} = S_i(\Omega)\Big|_{z=l+L_s^{(1)}+4L_f}}$$



Outlook & references

- Quantum limits of superresolution

Space



Time

To be done …

- References:

1. B H. Kolner, IEEE J. Quant. Electron. **30**, 1951(1994)
2. R. Salem, M. A. Foster, A. L. Gaeta, Adv. Opt. Phot. **5**, 274 (2013)
3. G. Patera and M. I. Kolobov, Opt. Lett. **40**, 1125 (2015)
4. G. Patera, J. Shi, D. B. Horoshko, and M. I. Kolobov, J. Opt. **19**, 054001(2017)